

A Proposal for Constraining Initial Vacuum by CMB

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Abstract. We propose a theoretical framework that can possibly constrain the initial vacuum by CMB. With a generic vacuum without any particular choice a priori, thereby keeping both the Bogolyubov coefficients in the analysis, we compute observable parameters from two- and three-point correlation functions. We are thus left with constraining four model parameters from the two complex Bogolyubov coefficients. We also demonstrate a method of finding out the constraint relations between the Bogolyubov coefficients using the theoretical normalization condition and observational data of power spectrum and bispectrum from CMB. Finally, we discuss the possible pros and cons of the analysis.

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1 Introduction

Inflationary paradigm is by now an widely accepted proposal that explains quite meticulously the early universe as well as the formation of seeds of classical perturbations. In this paradigm, the different modes of quantum fluctuations of initial vacuum, after horizon crossing, appear as classical perturbations and are measured as different observable parameters in Cosmic Microwave Background (CMB) observations [1–4]. However, even after the huge amount of progress both in the theoretical and the observational sectors, certain fundamental questions on the inflationary scenario are still haunting us. First, we do not know exactly who drives inflation. This is mostly because of the fact that several class of inflationary models still pass the observational tests, no matter how precise the latest observational results are (for an exhaustive review, see, for example, [5]). One can at best find out a class of *most probable* models using the model selection algorithm based on the Bayesian information criteria [6]. Even after that, we are left with a number of models with future add-ons to existing models.

On top of that, as the CMB temperature fluctuations are the effects of initial quantum fluctuation, hence from the CMB data we should, in principle, predict the nature of the primordial quantum fluctuations. However, the exact nature of this initial vacuum which plays a pivotal role in providing seeds of classical perturbations and hence observable parameters, is still eluding us. As it appears, a wide class of initial conditions fit the observational results pretty well. The widely accepted initial condition is the Bunch-Davies vacuum [7] (BD henceforth) analogous to Minkowski vacuum which is strictly valid only for field theory in flat space. It has been shown in some earlier literature that for curved space one gets infinite number of minimum energy states [7, 8] and the field is in principle allowed to choose any one of them as its vacuum depending on physical situation. Yet there are several other motivations of considering non-Bunch-Davies (NBD henceforth) vacuum as initial condition.

Some such examples can be found in [9–14]. For example, small periodic features in an inflation model can perturb the inflaton away from its BD state [10]. Also, the short distance modification of inflation, i.e., the effects of high energy cutoff can affect the initial condition [13]. Multifield dynamics can create excited state [14]. In the paper [11] considering an effective field theoretic motivation they gave a weak bound too on $|\beta| < 10^{-2}$, where β is the Bogolyubov coefficients. Some earlier, interesting attempts of constraining initial vacuum using different initial conditions and power spectrum alone can be found in [9]. However, we refrain ourselves from focusing on any particular mechanism or particular initial condition as such. We would rather try to bring the attention of the reader to the fact that from theoretical point of view, both BD and NBD vacua have sufficient motivations to invoke. However, as it turns out, the calculations of higher order perturbations are horrible for NBD vacua due to the presence of two Bogolyubov coefficients which can be complex quantities in general. So, to keep life simple, most of the case people still rely on the BD initial condition so far as the detailed perturbations are considered.

On the other hand, we are yet to comment conclusively on the statistical distribution of matter that originates from a fundamental theory, e.g., from inflationary paradigm. CMB data, both WMAP and Planck, have helped us measure the power spectrum and its scale dependence quite accurately but the uncertainties associated to the measurement of the observable parameter from three-point correlation function are huge even from the latest dataset of Planck 2015 [2–4] and includes the null value as well. A huge amount of work has been done on non-Gaussian predictions for specific models of inflation [15–37]. Although present observation suggests that the distribution is largely Gaussian, i.e, the stochastic properties are mostly determined by the two-point correlation function, and we are yet to find out any considerable departure from Gaussian nature of the primordial fluctuations, still primordial non-Gaussianity, even though of small amount, are crucial parameters to search for.

In this article we do a generic perturbation calculation without considering any particular vacuum, thereby keeping both α and β in the analysis and hence without considering Bunch-Davies vacuum a priori. We would rather leave it to the observational data to comment on them and try to constrain the allowed parameter space for those parameters from the available data Planck 2015. This will in turn constrain the choice of vacuum in the theory of inflation. Secondly, they are in principle complex quantities, which means there are actually four arbitrary parameters that need to be determined from initial conditions. So in principle, one needs to provide four different pieces of information, either from theoretical consideration or from observational constraints, to determine them completely. However, background theory and CMB data together provide three such information: (i) the normalization condition, (ii) the power spectrum and (iii) the Bispectrum, that too with huge errors for the third one. So, in principle, it is almost impossible to determine the initial vacuum from CMB data alone. Even with that, one can proceed to a great extent and constrain the parameter space determining the initial vacuum using those information. However, as it will turn out, the most challenging job towards this goal is to calculate the two- and three-point correlation functions and the observables therefrom for a generic vacuum in a model-independent way. So, in this article, we will mostly concentrate on the theoretical side, that is on calculation of those parameters for a generic vacuum. We will of course, demonstrate to some extent how one can in principle constrain the parameters space of initial vacuum using those information. This is only the first step in this direction. A rigorous numerical analysis using other datasets along with CMB that includes calculation of error in the parameters as well is in progress [38] and will be reported in due course.

Some discussions on the present work are in order. There are a handful of articles in the literature that explore non-Gaussianities for non-Bunch-Davies vacuum. Some such interesting works can be found in, say [39–46]. While those works are indeed intriguing and they form the background of our work, the objective of the present article is rather different. Precisely, we have two-fold goals: first, to show a calculation of non-Gaussianities for non-Bunch-Davies vacuum, which is as generic as possible, and secondly, to demonstrate how far we can constrain the parameter space of the vacuum directly from CMB observations. In fact, the second one is our major goal. Most of the previous articles dealing with non-Gaussianities for non-Bunch-Davies vacuum have either some or all of the following characteristics: they (i) consider only selective terms of the third order action,[36] (ii) consider only de Sitter solution for Mukhanov-Sasaki equation, (iii) calculate results by putting particular limits, e.g., squeezed limit, a priori (iv) attempt to constrain parameters from back-reaction which is not an observable. Our calculations are more generic in the sense that we (i) taking to account contributions from *all* terms of the third order action [36] that makes the analysis self-consistent (which is crucial if one attempts to constrain the parameters from observation), (ii) use quasi-de Sitter solutions of Mukhanov-Sasaki equation, which is more accurate than usually employed de Sitter solution, so far as latest observation is concerned (iii) calculate non-Gaussian parameters (Bispectrum and f_{NL}) for a generic vacuum and with all possible k 's, without confining ourselves to any particular shape a priori (we finally put different shapes only to confront with observations) (iv) attempt to constrain parameters directly from observations, which is more appealing. It should be mentioned here that strictly speaking, α and β are scale dependant and, in principle, one has to take into account their scale dependance in calculations. However, as argues in some of the previous articles, (say, for example, [47]), one does not expect much deviation from Bunch-Davies. Hence, instead of making the already complicated calculations more messy, α and β can roughly be considered as constants (they are, in fact, their average values), without major conflict with current observations. Since in this article, we were mostly bothered about giving a rough constraint to the parameters from present observations, we have safely considered them as constants.

The plan of the paper is as follows: In Section 2, we will do a brief review of the mode function calculations from second order perturbation of the action. Section 3 is mostly dedicated to the third order perturbation and calculation of bispectrum therefrom for a generic vacuum keeping both α and β terms. In the next two section, we show the corresponding expressions for the variance in bispectrum, i.e. f_{NL} for a generic vacuum followed by its values for different limiting cases. In Section 5, we compare our analytical results with the latest observable data from Planck 2015 and find out the possible bounds on the parameters, thereby putting constraints on the initial vacuum. We end up with a summary and possible future directions. The details of the calculations using Green's function method are given in Appendix A and B.

2 Review of Second Order Perturbation and Power Spectrum

For completeness, let us do a brief review of the inflationary dynamics and mode function calculations for second order perturbation consistent with the notations we use in this article. Some of the results from this section will also help us in doing numerical analysis in Section 5.

In what follows we will mostly concentrate on the Einstein-Hilbert action with canonically normalized scalar field minimally coupled to gravity and an arbitrary potential [8, 36,

52, 54, 55, 57]

$$S = \frac{1}{2} \int d^4x \sqrt{-g} [M_{pl}^2 R - (\nabla\phi)^2 - 2V(\phi)] \quad (2.1)$$

where $M_{pl} = (8\pi G)^{-\frac{1}{2}}$, R is Ricci curvature, ϕ is the inflaton field, and $V(\phi)$ the inflaton potential. Barring some non-canonical field(s) and non-minimal coupling, this action is fairly general and can represent a wide class of inflationary models [8, 36, 48, 49, 52, 54, 56, 57].

The background metric is as usual the FRW metric but with a somewhat unusual notation [36]

$$ds^2 = -dt^2 + e^{2\rho(t)} \delta_{ij} dx^i dx^j \quad (2.2)$$

where $e^{\rho(t)}$ is the scale factor. This unusual notation for the scale factor will help the complicated expressions (for the parameters derived from perturbative action) look a bit simplified. In this notation, the Hubble parameter just looks like

$$H(t) = \dot{\rho}(t) \quad (2.3)$$

and the background Friedmann equations and Klein-Gordon equation turn out to be

$$3M_{pl}^2 \dot{\rho}^2 = \frac{1}{2} \dot{\phi}^2 + V \quad (2.4)$$

$$\ddot{\rho} = -\frac{1}{2M_{pl}^2} \dot{\phi}^2 \quad (2.5)$$

$$\ddot{\phi} + 3\dot{\rho}\dot{\phi} + \frac{\partial V}{\partial \phi} = 0 \quad (2.6)$$

Consequently, the slow roll parameters take the form

$$\epsilon = -\frac{\ddot{\rho}}{\dot{\rho}^2} = \frac{1}{2} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^2} \quad ; \quad \eta = -\frac{\ddot{\phi}}{\dot{\rho}\dot{\phi}} \quad (2.7)$$

We will make frequent use of this form of ϵ .

The order-by-order perturbation of the background action (2.1) gives rise to the observable parameters. The primordial perturbation techniques for inflationary models are well discussed in [36] and reviewed in [8, 49, 50, 52, 57] and are very well known in the community. As mentioned, we will do a brief review of the mode function calculations for the sake of subsequent sections.

Parameterizing the scalar fluctuations in terms of the gauge invariant variable ζ [51], considering the comoving gauge in which there is no fluctuation in the scalar field, $\delta\phi = 0$ and expanding the action (2.1) to second order gives [36]

$$S^{(2)} = \frac{1}{2} \int d^4x [e^{3\rho} \frac{\dot{\phi}^2}{\dot{\rho}^2} \dot{\zeta}^2 - e^{\rho} \frac{\dot{\phi}^2}{\dot{\rho}^2} \partial_k \zeta \partial^k \zeta] \quad (2.8)$$

As is well known, the above action can be recast in terms of the Mukhanov variable $v = z\zeta$ (where $z = e^{\rho} \frac{\dot{\phi}}{\dot{\rho}}$) as

$$S^{(2)} = \frac{1}{2} \int d\tau d^3\vec{x} [(v')^2 - (\partial_i v)^2 + \frac{z''}{z} v] \quad (2.9)$$

where primes denote derivatives with respect to conformal time τ . Quantizing the Mukhanov variable v

$$\hat{v}(\tau, \vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^3} [v_k(\tau) \hat{a}_{\vec{k}} e^{i\vec{k} \cdot \vec{x}} + v_k^*(\tau) \hat{a}_{\vec{k}}^\dagger e^{-i\vec{k} \cdot \vec{x}}] \quad (2.10)$$

and varying the second order action (2.9) one arrives at the Mukhanov-Sasaki equation

$$v_k''(\tau) + \left(k^2 - \frac{z''}{z}\right) v_k(\tau) = 0 \quad (2.11)$$

where

$$\frac{z''}{z} = \frac{\nu^2 - \frac{1}{4}}{\tau^2}. \quad (2.12)$$

For perfect de Sitter universe the parameter $\nu = 3/2$. But recent observations from Planck 2015 data confirms a spectral tilt ($n_s \neq 1$) at 5σ . So, strictly speaking, one can no longer put $\nu = 3/2$ in the above equation (2.12) and in subsequent calculations. Rather, one needs to consider a value for the parameter ν consistent with latest value for scalar spectral index (n_s) as obtained from Planck 2015, say.

This equation has an exact solution in terms of Hankel function of 1st and 2nd kind [12, 49, 52, 60]

$$v_k(\tau) = \sqrt{-\tau} [\alpha H_\nu^{(1)}(-k\tau) + \beta H_\nu^{(2)}(-k\tau)] \quad (2.13)$$

Here α and β are otherwise arbitrary constants (Bogolyubov coefficients) the values of which are determined from the initial conditions. From the Wronskian condition for the mode function $v_k(\tau)$ we have a relation between α and β

$$|\alpha|^2 - |\beta|^2 = 1 \quad (2.14)$$

One should note that the above condition is a generic one which must be satisfied irrespective of the choice of vacuum. We will use the above relation later on as the first, and generic, information to evaluate α and β .

A brief discussion on α and β is in order. The widely accepted initial condition is Bunch-Davies (BD) initial condition with $\alpha = 1$ and $\beta = 0$. However, this initial condition is analogous to Minkowski vacuum which is strictly valid only for field theory in flat space. However, as pointed out earlier, there are enough theoretical motivations to consider vacua other than BD. For any other choice of the vacuum apart from BD, (NBD henceforth) α and β take more complicated values. This leads to somewhat arbitrariness in the calculation of observable parameters and also makes the calculation of perturbations a horrible task. So, to keep life simple, most of the cases people still rely on the Bunch-Davies initial condition. However a generic calculation for two point correlation function from the above second order perturbations using generic vacuum is already there in the literature. Below is the brief review of the same.

The power spectrum of the comoving curvature perturbation ζ can now be readily calculated from

$$P_k = |\zeta(k, \tau)|^2 = \frac{1}{z^2} |v_k(\tau)|^2 \quad (2.15)$$

Using the mode function equation (2.13) considering both α and β one gets,

$$P_k = \frac{(-\tau)}{z^2} [(|\alpha|^2 + |\beta|^2) |H_\nu^{(1)}|^2 + 2\text{Re}(\beta^* \alpha) (H_\nu^{(1)})^2] \quad (2.16)$$

Defining dimensionless power spectrum

$$\Delta_s^2 = \frac{k^3}{2\pi^2} P_k \quad (2.17)$$

and using the boundary condition for Hankel function

$$\lim_{k\tau \rightarrow 0} H_\nu^{(1)} = \frac{i}{\pi} \Gamma(\nu) \left(-\frac{k\tau}{2}\right)^{-\nu}$$

which is applicable for the superhorizon scales when the modes are well outside the horizon, the dimensionless power spectrum for super horizon limit $|k\tau| \ll 1$ takes the form

$$\Delta_s^2 = \frac{(-1)^{4-4\nu} \{\Gamma(\nu)\}^2 H^2 (1+\epsilon)^{1-2\nu}}{2^{2-2\nu} \pi^4 M_{pl}^2 \epsilon} \left(\frac{k}{aH}\right)^{3-2\nu} [|\alpha|^2 + |\beta|^2 - 2\text{Re}(\alpha\beta^*)]. \quad (2.18)$$

It is worthwhile to keep in mind the difference from BD vacuum (for which $\alpha = 1$ and $\beta = 0$ a priori).

The above relation (2.18) serves as the second piece of information and we will make use of its latest observational value from Planck 2015 to constrain the initial vacuum.

3 Third order perturbation and Bispectrum

As obvious from the last section the power spectrum for generic vacuum and for quasi-de Sitter solution is given by equation (2.18). In this article, our primary intention is to carry forward these theoretical calculations to third order perturbations without any particular choice of vacuum and for quasi-de Sitter mode function. Consequently we will keep both α and β in our analysis of third order perturbations. We remind the interested reader that our calculation is more generic than the existing literature on non-Bunch-Davies vacuum for either of the four reasons pointed out in the Introduction (Section 1).

Nevertheless, we will further employ observational data to comment on the Bogolyubov coefficients and try to give a crude constraint on those parameters from the available data. This will in turn constrain the choice of vacuum in the theory of inflation.

However, since α and β are complex quantities, which means there are actually four arbitrary parameters that need to be determined from initial conditions. So in principle, one needs to provide four different pieces of information, either from theoretical consideration or from observational constraints, to determine them completely. The previous section gives two of them: (i) the normalization condition and (ii) the two point correlation function i.e., the power spectrum for a generic vacuum. In this section we intend to search for possible information from three point correlation function and hence from bispectrum.

3.1 Interaction Hamiltonian and three-point Correlation Function

In order to find out the bispectrum for a generic vacuum, we first need to expand the action (2.1) to third order [8, 36, 52, 57]

$$\begin{aligned} S^{(3)} = & \int d^4x \left[\frac{1}{4} \frac{e^{3\rho}}{M_{pl}^2} \frac{\dot{\phi}^4}{\dot{\rho}^4} \dot{\zeta}^2 \zeta + \frac{1}{4} \frac{e^\rho}{M_{pl}^2} \frac{\dot{\phi}^4}{\dot{\rho}^4} \zeta \partial_k \zeta \partial^k \zeta - \frac{1}{2} \frac{e^{3\rho}}{M_{pl}^2} \frac{\dot{\phi}^4}{\dot{\rho}^4} \dot{\zeta} \partial_k \zeta \partial^k (\partial^{-2} \dot{\zeta}) \right. \\ & \left. + \frac{1}{2} e^{3\rho} \frac{\dot{\phi}^2}{\dot{\rho}^2} \dot{\zeta} \zeta^2 \frac{d}{dt} \left[\frac{\ddot{\phi}}{\dot{\phi} \dot{\rho}} + \frac{1}{2} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^2} \right] - \frac{1}{16} \frac{e^{3\rho}}{M_{pl}^4} \frac{\dot{\phi}^6}{\dot{\rho}^6} [\dot{\zeta}^2 \zeta - \zeta \partial_k \partial_l (\partial^{-2} \dot{\zeta}) \partial^k \partial^l (\partial^{-2} \dot{\zeta})] \right] \end{aligned}$$

$$+f(\zeta)\left\{\frac{d}{dt}[e^{3\rho}\frac{\dot{\phi}^2}{\dot{\rho}^2}\dot{\zeta}]-e^\rho\frac{\dot{\phi}^2}{\dot{\rho}^2}\partial_k\partial^k\zeta\right\} \quad (3.1)$$

where the function $f(\zeta)$ has the following form

$$\begin{aligned} f(\zeta) = & \frac{1}{2}\left[\frac{\ddot{\phi}}{\dot{\phi}\dot{\rho}} + \frac{1}{2}\frac{1}{M_{pl}^2}\frac{\dot{\phi}^2}{\dot{\rho}^2}\right]\zeta^2 + \frac{1}{\dot{\rho}}\dot{\zeta}\zeta - \frac{1}{4}\frac{e^{-2\rho}}{\dot{\rho}^2}[\partial_k\zeta\partial^k\zeta - \partial^{-2}\partial_k\partial_l(\partial^k\zeta\partial^l\zeta)] \\ & + \frac{1}{4}\frac{1}{M_{pl}^2}\frac{\dot{\phi}^2}{\dot{\rho}^3}[\partial_k\zeta\partial^k(\partial^{-2}\dot{\zeta}) - \partial^{-2}\partial_k\partial_l(\partial^k\zeta\partial^l(\partial^{-2}\dot{\zeta}))] \end{aligned} \quad (3.2)$$

At the very first place, the above action looks quite difficult to handle. However, it can be reduced to a much simplified form after a field redefinition of the gauge invariant variable ζ as following

$$\zeta = \zeta_n + f(\zeta_n).$$

With this the above third order perturbed action takes the following form

$$S^{(3)}(\zeta_n) = \int d^4x \frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^4}{\dot{\rho}^4} \{e^{3\rho}\dot{\zeta}_n^2\zeta_n + e^\rho\zeta_n\partial_k\zeta_n\partial^k\zeta_n - 2e^{3\rho}\dot{\zeta}_n\partial_k\zeta_n\partial^k(\partial^{-2}\dot{\zeta}_n)\} \quad (3.3)$$

We are now in a position to explore the third order perturbative action (3.3) and calculate the three point correlation functions. In what follows we will employ the Schwinger-Keldysh in-in formalism [49, 52, 57, 58] for computation. In this method, the interaction Hamiltonian can be readily written from the action (3.3) as:

$$H_I(t) = - \int d^3\vec{x} \frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^4}{\dot{\rho}^4} \{e^{3\rho}\dot{\zeta}_n^2\zeta_n + e^\rho\zeta_n\partial_k\zeta_n\partial^k\zeta_n - 2e^{3\rho}\dot{\zeta}_n\partial_k\zeta_n\partial^k(\partial^{-2}\dot{\zeta}_n)\} \quad (3.4)$$

This interaction Hamiltonian will be used to evaluate the three point function as follows. We first expanded the three point function as

$$\begin{aligned} & \langle 0(t) | \zeta(t, \vec{x}) \zeta(t, \vec{y}) \zeta(t, \vec{z}) | 0(t) \rangle \\ = & \langle 0(t) | \zeta_n(t, \vec{x}) \zeta_n(t, \vec{y}) \zeta_n(t, \vec{z}) | 0(t) \rangle + \langle 0(t) | \zeta_n(t, \vec{x}) \zeta_n(t, \vec{y}) f(\zeta_n(t, \vec{z})) | 0(t) \rangle \\ & + \langle 0(t) | \zeta_n(t, \vec{x}) f(\zeta_n(t, \vec{y})) \zeta_n(t, \vec{z}) | 0(t) \rangle + \langle 0(t) | f(\zeta_n(t, \vec{x})) \zeta_n(t, \vec{y}) \zeta_n(t, \vec{z}) | 0(t) \rangle \\ & + \dots \end{aligned}$$

Here the dotted term represent Higher Order terms that include at least five features of the Shifted function. They are suppressed due to the higher order slow roll parameter effects. Note that the three point function $\zeta_n(t, \vec{x})$ would vanish entirely if evaluated in the initial state and in interaction picture its value entirely generated by the evolution of state.

Thus, in terms of the interaction Hamiltonian, the three point correlation function takes the form

$$\begin{aligned} & \langle 0(t) | \zeta_n(t, \vec{x}) \zeta_n(t, \vec{y}) \zeta_n(t, \vec{z}) | 0(t) \rangle \\ = & \langle 0 | (T e^{-i \int_{-\infty}^t dt' H_I(t')})^\dagger \zeta_n(t, \vec{x}) \zeta_n(t, \vec{y}) \zeta_n(t, \vec{z}) (T e^{-i \int_{-\infty}^t dt' H_I(t')}) | 0 \rangle \end{aligned}$$

For the rest of the terms as $f(\zeta_n)$ are quartic in the field ζ_n , they do contribute in initial state $|0\rangle$. In this case time evolution of the state only contributes at a higher order in slow roll parameter.

One is now left with the task of inserting the interaction Hamiltonian (3.4) into the above expression of three point correlation function and calculate the effects of each term using Green's function method. However, one needs to keep in mind that everything has to be calculated for a generic vacuum using the mode function (2.13). This turns out to be quite a daunting task, first, due to the form of the interaction Hamiltonian and, secondly, because of the presence of both α and β in the mode function.

The detailed calculations for this particular action are given in *Appendix A*. In the next sections, we will make use of those results in the calculation of bispectrum and subsequently in f_{NL} , which will finally be subject to the constraints set by Planck 2015.

3.2 Bispectrum for generic vacuum

The leading order non-Gaussianity is given by the Fourier transform of three point Correlation function usually known as Bispectrum, which is defined as [2, 8]

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{m}} \zeta_{\mathbf{n}} \rangle = B_{\zeta}(\mathbf{k}, \mathbf{m}, \mathbf{n}) = (2\pi)^3 \delta(\mathbf{k} + \mathbf{m} + \mathbf{n}) B_{\zeta}(k, m, n) \quad (3.5)$$

Now to calculate the three point function or bispectrum we will evaluate the following integration following [36, 49, 52, 57]

$$\langle 0(t) | \zeta(t, \vec{x}) \zeta(t, \vec{y}) \zeta(t, \vec{z}) | 0(t) \rangle = -i \int_{-\infty}^t \langle 0 | [\zeta(t, \vec{x}) \zeta(t, \vec{y}) \zeta(t, \vec{z}), H_I] | 0 \rangle \quad (3.6)$$

using the interaction Hamiltonian (3.4) and the results given in *Appendix A*. Let us remind the reader that usually the calculation of three point correlation function and bispectrum are done considering the BD vacuum for which $\alpha = 1$ and $\beta = 0$. This makes the calculations easier to handle. On the contrary, for a generic vacuum without any preferred value of α and β , as earlier, the detailed calculations are quite complicated due to the presence of both α and β in the mode function (2.13) and one needs to consider the contribution from each term separately.

However, we have been able to find out the bispectrum for the generic case. For interested readers, the details are given in *Appendix B*. In this section we will just take up the final result for bispectrum that reads

$$B_{\zeta}(k, m, n) = (2\pi)^3 \delta^3(\vec{k} + \vec{m} + \vec{n}) \frac{1}{k^3 m^3 n^3} \frac{H^4}{32\epsilon^2 M_{pl}^4} \mathcal{A} \quad (3.7)$$

For subsequent calculations of f_{NL} , the only complicated expression we explicitly show here is that of the function \mathcal{A} :

$$\begin{aligned} \mathcal{A} = & \left\{ \left(\frac{\ddot{\phi}}{\dot{\phi}\dot{\rho}} + \frac{1}{2} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^2} \right) A^2 + \frac{2}{\dot{\rho}} AB - \frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^3} AB + \left(-i \frac{2\epsilon^2 M_{pl}^2}{H^4 (1-\epsilon)^4} \right) D_1 \right\} \frac{32\epsilon^2 M_{pl}^4}{H^4} (k^3 + m^3 + n^3) \\ & + \left[\frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^3} \frac{32\epsilon^2 M_{pl}^4}{H^4} AB \left\{ \frac{(k^3 - m^3)(k^2 - m^2)}{2n^2} + \frac{(k^3 - n^3)(k^2 - n^2)}{2m^2} + \frac{(m^3 - n^3)(m^2 - n^2)}{2k^2} \right\} \right] \end{aligned}$$

$$\begin{aligned}
& -\frac{k}{4}(k^2+m^2-n^2)(k^2-m^2+n^2)(\frac{1}{m^2}+\frac{1}{n^2})-\frac{m}{4}(k^2+m^2-n^2)(m^2-k^2+n^2)(\frac{1}{k^2}+\frac{1}{n^2}) \\
& -\frac{n}{4}(k^2+n^2-m^2)(m^2-k^2+n^2)(\frac{1}{k^2}+\frac{1}{m^2})\} + [\frac{i128\epsilon^4 M_{pl}^6}{(1-\epsilon)^4 H^8}][\frac{kmn(km+kn+mn)}{(k+m+n)^2}D_1 \\
& +\{\frac{km}{(k+m-n)^2}+\frac{kn}{(k+n-m)^2}+\frac{mn}{(m+n-k)^2}\}kmnD_2+\{\frac{km}{(m+n-k)^2}+\frac{km}{(k+n-m)^2} \\
& +\frac{kn}{(m+n-k)^2}+\frac{kn}{(k+m-n)^2}+\frac{mn}{(k+m-n)^2}+\frac{mn}{(k+n-m)^2}\}kmn(D_3+D_4)] \\
& +[-i\frac{128\epsilon^4 M_{pl}^6}{(1-\epsilon)^2 H^6}][\{-\frac{1}{2}\frac{(k^2+m^2+n^2)}{(k+m+n)}\}kmn(C_3^3+C_4^3)-\frac{1}{2}\{\frac{(k^2+m^2+n^2)}{(k+m-n)^2}+\frac{(k^2+m^2+n^2)}{(k+n-m)^2} \\
& +\frac{(k^2+m^2+n^2)}{(m+n-k)^2}\}C_3C_4(C_3+C_4)kmn]+[-\frac{i128\epsilon^4 M_{pl}^6}{(1-\epsilon)^4 H^8}][-\{\frac{1}{2}\frac{(k^4+m^4+n^4)}{(k+m+n)}+\frac{k^2m^2+k^2n^2+m^2n^2}{(k+m+n)} \\
& +kmn\frac{(k^2+m^2+n^2)}{(k+m+n)}\}D_1+\{\frac{n}{2}\frac{((k^2-m^2)^2-n^2(k^2+m^2))}{(k+m-n)^2}+\frac{m}{2}\frac{((k^2-n^2)^2-m^2(k^2+n^2))}{(k+n-m)^2} \\
& +\frac{k}{2}\frac{((m^2-n^2)^2-k^2(m^2+n^2))}{(-k+m+n)^2}\}D_2+\{\frac{n}{2}((k^2-m^2)^2-n^2(k^2+m^2))(\frac{1}{(m+n-k)^2}+\frac{1}{(k+n-m)^2}) \\
& +\frac{m}{2}((k^2-n^2)^2-m^2(k^2+n^2))(\frac{1}{(m+n-k)^2}+\frac{1}{(k+m-n)^2}) \\
& +\frac{k}{2}((m^2-n^2)^2-k^2(m^2+n^2))(\frac{1}{(k+m-n)^2}+\frac{1}{(k+n-m)^2})\}(D_3+D_4)] \quad (3.8)
\end{aligned}$$

In the above, the information of the mode function and hence the vacuum is encoded in the expressions of the quantities A , B , C_3 , C_4 , D_1 , D_2 , D_3 and D_4 . This will be obvious if we write them down explicitly:

$$\begin{aligned}
A &= \frac{H^2(4\nu^2-1)^2(\epsilon-1)^2}{64\pi\epsilon M_{pl}^2}[|\alpha|^2+|\beta|^2-ie^{i\nu\pi}\alpha^*\beta+ie^{-i\nu\pi}\beta^*\alpha] \\
B &= -(\epsilon-1)^2\frac{H^3(4\nu^2-1)^2}{32\pi M_{pl}^2}[|\alpha|^2+|\beta|^2-i\alpha^*\beta e^{i\nu\pi}+i\beta^*\alpha e^{-i\nu\pi}]
\end{aligned}$$

$$\begin{aligned}
C_3 &= \frac{-iH^2(4\nu^2 - 1)(\epsilon - 1)^2}{8\pi\epsilon M_{pl}^2} [|\alpha|^2 - i\alpha^*\beta e^{i\nu\pi}] \\
C_4 &= \frac{-iH^2(4\nu^2 - 1)(\epsilon - 1)^2}{8\pi\epsilon M_{pl}^2} [-|\beta|^2 - i\alpha\beta^* e^{-i\nu\pi}] \\
C_7 &= \frac{H^3(4\nu^2 - 1)(\epsilon - 1)^3}{8\pi\epsilon M_{pl}^2} [|\alpha|^2 - i\alpha^*\beta e^{i\nu\pi}] \\
C_8 &= \frac{H^3(4\nu^2 - 1)(\epsilon - 1)^3}{8\pi\epsilon M_{pl}^2} [|\beta|^2 + i\alpha\beta^* e^{-i\nu\pi}]
\end{aligned}$$

$$D_1 = C_3 C_7^2 + C_4 C_8^2, \quad D_2 = C_3 C_8^2 + C_4 C_7^2, \quad D_3 = C_3 C_7 C_8, \quad D_4 = C_4 C_7 C_8$$

It is a straightforward exercise to check that one gets back the usual results for BD vacuum if one considers both α and β are real numbers and $\alpha = 1, \beta = 0$.

4 Calculation of f_{NL} from Bispectrum

Even though after painstaking exercise we have succeeded in finding out the bispectrum for a generic vacuum with quasi-de Sitter mode function, if one really wants to constrain the vacuum using observations, one needs to proceed further and calculate its variance, namely, the f_{NL} . Of course, we have not yet detected non-Gaussianities, so any value of f_{NL} in the range predicted by Planck 2015 is allowed, we will show in the subsequent section how the present information alone can help us constrain the vacuum to some extent.

Using the expression from bispectrum from Eq (3.7), one can define its variance f_{NL} as [36]

$$-f_{NL} \sim \frac{5}{3} \frac{\mathcal{A}}{(4\sum_i k_i^3)} = \frac{5}{12} \frac{\mathcal{A}}{(k^3 + m^3 + n^3)} \quad (4.1)$$

With the expression of \mathcal{A} given in Eq (3.8) the f_{NL} we get

$$\begin{aligned}
-f_{NL} &= \frac{5}{12} \left[\left(\frac{\ddot{\phi}}{\dot{\phi}\dot{\rho}} + \frac{1}{2} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^2} \right) A^2 + \frac{2}{\dot{\rho}} AB - \frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^3} AB + \left(-i \frac{2\epsilon^2 M_{pl}^2}{H^4(1-\epsilon)^4} \right) D_1 \right] \frac{32\epsilon^2 M_{pl}^4}{H^4} \\
&+ \frac{5}{12} \left[\frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^3} \frac{32\epsilon^2 M_{pl}^2}{H^4} AB \left\{ \frac{1}{2k^2} \frac{(m^3 - n^3)(m^2 - n^2)}{(k^3 + m^3 + n^3)} + \frac{1}{2m^2} \frac{(k^3 - n^3)(k^2 - n^2)}{(k^3 + m^3 + n^3)} \right. \right. \\
&+ \frac{1}{2n^2} \frac{(k^3 - m^3)(k^2 - m^2)}{(k^3 + m^3 + n^3)} - \frac{k}{4} \frac{(k^2 + m^2 - n^2)(k^2 - m^2 + n^2)}{(k^3 + m^3 + n^3)} \left(\frac{1}{m^2} + \frac{1}{n^2} \right) \\
&- \frac{m}{4} \frac{(k^2 + m^2 - n^2)(m^2 - k^2 + n^2)}{(k^3 + m^3 + n^3)} \left(\frac{1}{k^2} + \frac{1}{n^2} \right) - \frac{n}{4} \frac{(k^2 + n^2 - m^2)(m^2 - k^2 + n^2)}{(k^3 + m^3 + n^3)} \left(\frac{1}{k^2} + \frac{1}{m^2} \right) \Big\} \\
&+ \left. i \frac{128\epsilon^4 M_{pl}^6}{(1-\epsilon)^4 H^8} \right\} \left\{ \frac{kmn(km + kn + mn)}{(k^3 + m^3 + n^3)(k + m + n)^2} D_1 + \left\{ \frac{km}{(k + m - n)^2} + \frac{kn}{(k + n - m)^2} \right\} \right\}
\end{aligned}$$

$$\begin{aligned}
& + \frac{mn}{(m+n-k)^2} \} \frac{kmnD_2}{(k^3+m^3+n^3)} + \left\{ \frac{km}{(m+n-k)^2} + \frac{km}{(k+n-m)^2} + \frac{kn}{(m+n-k)^2} \right. \\
& + \frac{kn}{(k+m-n)^2} + \frac{mn}{(k+m-n)^2} + \left. \frac{mn}{(k+n-m)^2} \right\} \frac{kmn(D_3+D_4)}{(k^3+m^3+n^3)} \\
& + [-i \frac{128\epsilon^4 M_{pl}^6}{(1-\epsilon)^2 H^6}] \left[\left\{ -\frac{1}{2} \frac{(k^2+m^2+n^2)}{(k+m+n)^2} \right\} \frac{kmn}{(k^3+m^3+n^3)} (C_3^3+C_4^3) \right. \\
& - \frac{1}{2} \left\{ \frac{(k^2+m^2+n^2)}{(k+m-n)^2} + \frac{(k^2+m^2+n^2)}{(k+n-m)^2} + \frac{(k^2+m^2+n^2)}{(m+n-k)^2} \right\} \frac{kmn}{(k^3+m^3+n^3)} C_3 C_4 (C_3+C_4) \Big] \\
& + \left\{ -\frac{i128\epsilon^4 M_{pl}^6}{(1-\epsilon)^4 H^8} \right\} \left[-\left\{ \frac{1}{2} \frac{(k^4+m^4+n^4)}{(k+m+n)(k^3+m^3+n^3)} + \frac{k^2 m^2 + k^2 n^2 + m^2 n^2}{(k+m+n)(k^3+m^3+n^3)} \right. \right. \\
& + \frac{kmn(k^2+m^2+n^2)}{(k+m+n)^2(k^3+m^3+n^3)} \Big\} D_1 + \left\{ \frac{n}{2} \frac{((k^2-m^2)^2 - n^2(k^2+m^2))}{(k+m-n)^2(k^3+m^3+n^3)} + \frac{m}{2} \frac{((k^2-n^2)^2 - m^2(k^2+n^2))}{(k+n-m)^2(k^3+m^3+n^3)} \right. \\
& + \frac{k}{2} \frac{((m^2-n^2)^2 - k^2(m^2+n^2))}{(m+n-k)^2(k^3+m^3+n^3)} \Big\} D_2 + \left\{ \frac{n}{2} \frac{((k^2-m^2)^2 - n^2(k^2+m^2))}{(k^3+m^3+n^3)} \left(\frac{1}{(m+n-k)^2} + \frac{1}{(k+n-m)^2} \right) \right. \\
& + \frac{m}{2} \frac{((k^2-n^2)^2 - m^2(k^2+n^2))}{(k^3+m^3+n^3)} \left(\frac{1}{(m+n-k)^2} + \frac{1}{(k+m-n)^2} \right) \\
& + \left. \frac{k}{2} \frac{((m^2-n^2)^2 - k^2(m^2+n^2))}{(k^3+m^3+n^3)} \left(\frac{1}{(k+m-n)^2} + \frac{1}{(k+n-m)^2} \right) \right\} (D_3+D_4) \Big] \quad (4.2)
\end{aligned}$$

Below we will explore different shapes of mode to evaluate f_{NL} which are relevant for observational purpose [2].

4.1 f_{NL} in Squeezed limit

Here we considered squeezed limit case where one of the modes is very small relative to the other two modes

$$\vec{k} \longrightarrow 0 \quad \vec{m} \approx -\vec{n} \quad \text{or} \quad m \approx n$$

Using the generic expression of from Eq (4.2) and applying the above condition we get

$$-f_{NL}^{\text{squeez}} = \frac{5}{12} \left[\left(\frac{\ddot{\phi}}{\dot{\phi}\dot{\rho}} + \frac{1}{2} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^2} \right) A^2 + \frac{2}{\dot{\rho}} AB - \frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^3} AB - \frac{2i\epsilon^2 M_{pl}^2}{H^4(1-\epsilon)^4} D_1 \right] \frac{32\epsilon^2 M_{pl}^4}{H^4}$$

$$\begin{aligned}
& + \frac{5}{12} \left[\frac{128i\epsilon^4 M_{pl}^6}{(1-\epsilon)^4 H^8} (D_3 + D_4) + \frac{64i\epsilon^4 M_{pl}^6}{(1-\epsilon)^4 H^8} D_1 \right. \\
& \left. + \left\{ \frac{128i\epsilon^4 M_{pl}^6}{(1-\epsilon)^4 H^8} (D_3 + D_4) + \frac{128i\epsilon^4 M_{pl}^6}{(1-\epsilon)^2 H^6} C_3 C_4 (C_3 + C_4) \right\} \frac{m}{k} \right] \quad (4.3)
\end{aligned}$$

Note that unlike the BD case, there is a term with $\frac{m}{k}$ sitting in the expression for f_{NL} in Squeezed limit. This arises solely due to a nonzero β and the expression cannot be readily reduced to that of the BD case. The possible conclusions from the above expression for squeezed limit can be the following:

- Here we have got a new result that f_{NL} in squeezed limit is scale dependent as we can find that f_{NL} is having a $\frac{m}{k}$ term. So here we may conclude that for non Bunch-Davies vacuum we can get scale dependent squeezed limit value of f_{NL} for single field slow-roll model.
- Since in the squeezed limit, $k \rightarrow 0$, maybe future detection of f_{NL} in squeezed limit will set a scale of k and will answer how much 'squeezed' it can be.
- In the worst case, f_{NL} for squeezed limit cannot be estimated for any vacuum other than the Bunch-Davies one.

However, we must admit that all of these are mere speculations and we refrain ourselves from making a definite comment on this. We would rather make use of the f_{NL} in Equilateral limit calculated in the next subsection for constraining the initial vacuum.

4.2 f_{NL} in Equilateral limit

For another shape of modes, where all modes are equal in magnitude known as equilateral shape we calculated the f_{NL}

$$k = m = n$$

Similarly as done in squeezed limit case applying the above relation of modes in the generic expression for f_{NL} from Eq (4.2), we find

$$\begin{aligned}
-f_{NL}^{\text{equil}} = & \frac{5}{12} \left[\left(\frac{\ddot{\phi}}{\dot{\phi}\dot{\rho}} + \frac{1}{2} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^2} \right) A^2 + \frac{2}{\dot{\rho}} AB - \frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^3} AB - \frac{2i\epsilon^2 M_{pl}^2}{H^4 (1-\epsilon)^4} D_1 \right] \frac{32\epsilon^2 M_{pl}^4}{H^4} \\
& + \frac{5}{12} \left[-\frac{1}{8} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^3} \frac{32\epsilon^2 M_{pl}^4}{H^4} AB + \frac{128i\epsilon^4 M_{pl}^6}{(1-\epsilon)^4 H^8} \left\{ \frac{D_1}{9} + D_2 + 2(D_3 + D_4) \right\} \right. \\
& + \frac{128i\epsilon^4 M_{pl}^6}{(1-\epsilon)^2 H^6} \left\{ \frac{1}{18} (C_3^3 + C_4^3) + \frac{3}{2} C_3 C_4 (C_3 + C_4) \right\} \\
& \left. + \frac{128i\epsilon^4 M_{pl}^6}{(1-\epsilon)^4 H^8} \left\{ \frac{11}{18} D_1 + D_2 + 2(D_3 + D_4) \right\} \right] \quad (4.4)
\end{aligned}$$

where, as before, the quantities A , B , C_i and D_i contain all the information about a generic vacuum and hence about the parameters α and β .

f_{NL} in Equilateral limit in Eq (4.4) will serve as the complimentary piece of information along with the normalization condition Eq (2.14) and power spectrum Eq (2.18) to constrain the allowed parameter space for initial vacuum. In what follows we shall explore this in details.

5 Possible constraints on the parameters from Planck 2015

As pointed out earlier, for a generic vacuum, the parameters α and β are in principle complex quantities, which means there are actually four arbitrary parameters that need to be determined from initial conditions. So in principle, one needs to provide four different pieces of information, either from theoretical consideration or from observational constraints, to determine them completely. In this article we have explored this possibility and found that we can have three pieces of information from early universe and CMB:

- the normalization condition Eq (2.14) which is generic one and must be satisfied for any vacuum, irrespective of whether it is BD or NBD.
- the power spectrum for a generic vacuum (2.18) and its numerical value from Planck 2015
- information from the bispectrum, in particular, the f_{NL} in Equilateral limit in Eq (4.4) and its bound as given by Planck 2015.

Given that we are going to determine four unknown parameters but we are given only three pieces of information, we are thus left with the choice of finding out the allowed parameter space for α and β . On top of that, given that we have not yet detected primordial non-Gaussianities and Planck 2015 can at best give some bounds on f_{NL} , we have to deal with the uncertainty of having zero non-Gaussianity as well, thereby losing this third piece of information. We would rather be somewhat optimistic and take the bounds on f_{NL} at the face value. As it appears, this will indeed help us in constraining the allowed parameter space for α and β to some extent which will in turn put constraints on the choice of the vacuum.

Below we will find out the constrain relations between α and β from the above informations. The first constraint relation is straightforward and it is given by the normalization condition (2.14):

$$|\alpha|^2 - |\beta|^2 = 1 \quad (5.1)$$

From Eq (2.18), the amplitude of power spectrum for this generic vacuum is given by

$$A_s = \frac{(-1)^{4-4\nu} \{\Gamma(\nu)\}^2 H^2 (1+\epsilon)^{1-2\nu}}{2^{2-2\nu} \pi^4 M_{pl}^2 \epsilon} [|\alpha|^2 + |\beta|^2 - 2\text{Re}(\beta^* \alpha)] \quad (5.2)$$

As we have mentioned earlier for perfect de-Sitter universe the parameter $\nu = 3/2$. But recent observations from Planck 2015 data confirms a spectral tilt ($n_s \neq 1$) at 5σ . So, strictly speaking, one can no longer put $\nu = 3/2$ in the expressions for power spectrum, bispectrum and f_{NL} , as done in most of the existing literature. Rather, one needs to consider a value for the parameter ν consistent with latest value for scalar spectral index (n_s) as obtained from Planck 2015. This is precisely what we are going to do in the subsequent calculations. Our analysis is, therefore, more up-to-date and consistent with latest observations.

The value of scalar spectral index (n_s) as obtained from Planck 2015 is $n_s = 0.9677 \pm 0.0060$ (TT+lowP+lensing)[3]. In what follows, we will make use of the best fit value only. Using the relation between n_s and ν , $\nu = (2 - \frac{n_s}{2})$ we obtain the best fit value of the parameter $\nu = 1.5162$ from the best fit value of n_s , which we are going to use throughout.

Further, combined Planck TT + low p results give the best fit value for $\epsilon = 0.0068$ [95% CL] [3]. Combining all these, the amplitude of scalar power spectrum for a generic vacuum turns out to be

$$A_s = \frac{10^{-6}}{354.1012} [|\alpha|^2 + |\beta|^2 - 2\text{Re}(\beta^* \alpha)] \quad (5.3)$$

One can now compare this result with the Planck 2015 data [3]. From combined Planck TT + low p + lensing, we have $\ln(10^{10} A_s) = 3.062 \pm 0.029$ [68% CL] Strictly speaking, one should consider the best fit value along with the error bars in the analysis. However, as mentioned earlier, our primary intention in this paper is to demonstrate how one can utilize these pieces of information to constrain the parameters. So, keeping in mind that the error bars in power spectrum are small compared to that of bispectrum (which we will make use of in the next step), we can safely neglect the error in power spectrum and take into account its best fit value only. This will somewhat simplify our analysis. Using the best fit value, Eq (5.2) give rise to a constraint relation between α and β as

$$|\alpha|^2 + |\beta|^2 - 2\text{Re}(\beta^* \alpha) = 0.7567 \quad (5.4)$$

The third constraint relation comes from exploring the f_{NL} in Equilateral limit. Expressing explicitly in terms of α and β via the parameters A, B, C_i, D_i etc, and using the best fit value for $\eta_p = 0.029$ [68 % CL Planck TT + low p] [3] (where $-\eta = \frac{\eta_p}{2} - \epsilon$), along with the previously used values for ν and ϵ , the f_{NL}^{equil} takes the following relatively simple form

$$\begin{aligned} -f_{NL}^{equil} &= \frac{5}{12} \times (0.0049) [(21.0143)(|\alpha|^2 - \alpha^* \beta)(|\beta|^2 - \beta^* \alpha)(|\alpha|^2 - |\beta|^2 - \alpha^* \beta + \beta^* \alpha) \\ &\quad - (9.0225)(|\alpha|^2 + |\beta|^2 - \alpha^* \beta - \beta^* \alpha)^2] \end{aligned} \quad (5.5)$$

As is well known, the observational bounds for f_{NL} comes with huge errors. Also, the uncertainties include the zero non-Gaussianities as well. Of course, future observations can improve the scenario further and in principle, one can use the improved value of f_{NL} to constrain the vacuum following our analysis. At this moment, the best one can do is to take into account the huge errors in the analysis and search for the maximum possible information till date.

From Planck 2015 [2][68% CL temp and polarization] we have

$$f_{NL}^{equil} = -4 \pm 43 \quad (5.6)$$

which, when compared to the above expression, gives the third constraint relation between α and β as below

$$\begin{aligned} 21.0143(|\alpha|^2 - \alpha^* \beta)(|\beta|^2 - \beta^* \alpha)(|\alpha|^2 - |\beta|^2 - \alpha^* \beta + \beta^* \alpha) \\ - 9.0225(|\alpha|^2 + |\beta|^2 - \alpha^* \beta - \beta^* \alpha)^2 = -490.7976 \times (-4 \pm 43) \end{aligned} \quad (5.7)$$

As pointed out earlier, we are now left with these three constraint relations Eq (5.1), (5.4), (5.7) and four unknowns. As we committed before now we will try to extract the

maximum information about α and β from the above discussed three relations. For that purpose let us break the α and β in real and imaginary parts by considering

$$\alpha = a_1 + ia_2 \quad \beta = b_1 + ib_2 \quad (5.8)$$

where a_1, a_2, b_1, b_2 are the four unknowns that need to be constrained. Putting them back into the three constraint relations and subsequently, solving them numerically, we finally arrive at the allowed region of the parameters from Planck 2015.

The following figures (Fig 1-3) represent the allowed region for each pair of parameters. For each figure, we have chosen two parameters from the set $\{a_1, a_2, b_1, b_2\}$ and expressed them in terms of the other two. We have finally utilized the three constraint relations to find out the allowed regions for each pair numerically.

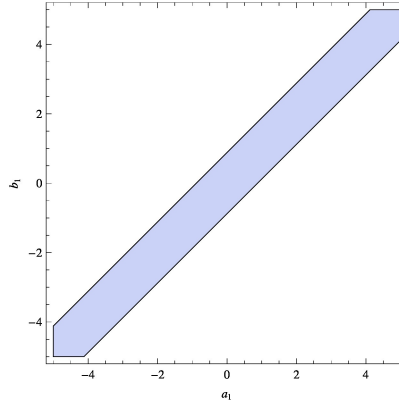


Figure 1. The figure shows the allowed region for a_1 vs b_1 plot. The Bunch-Davies (BD) vacuum, which is just a point $\{1, 0\}$ in this plot, sits comfortably inside with a spread allowing some NBD vacua as well.

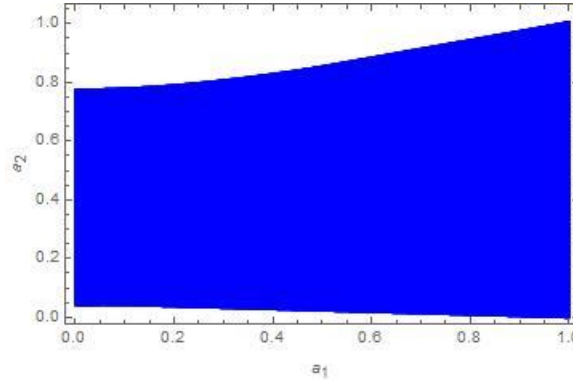


Figure 2. This figure depicts the allowed region for a_1 vs a_2 considering both BD and NBD vacuum.

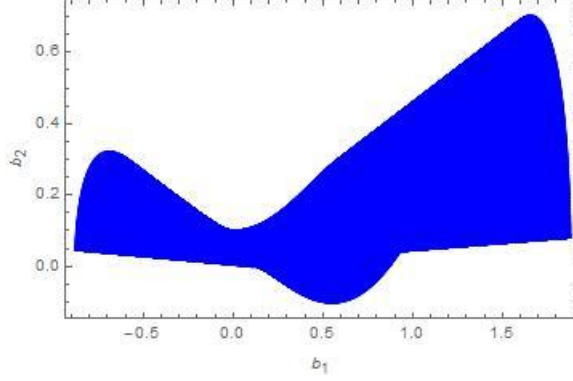


Figure 3. This figure represents the allowed region for b_1 vs b_2 considering both BD and NBD vacuum.

The above plots summarize our results and claim. What they essentially tell us is that the Bunch-Davies vacuum is still a well-acceptable vacuum as rightly done in most of the calculations. However, since we get a spread over the straight line (due to the uncertainties in non-Gaussian parameters), this essentially means quite a handful of non-Bunch-Davies vacua are equally allowed so far as the present data is concerned. Nevertheless, whenever one proposes a new vacuum, one need to check if it falls within the allowed region of CMB observations following the method demonstrated in the present article.

6 Summary, Limitations and Future Directions

In this article, we attempted to demonstrate how one can possibly constrain the initial vacuum using CMB. Our analysis is mostly a theoretical one which focuses on the calculation of observable parameters for a generic vacuum (i.e., without any particular choice of the vacuum a priori), thereby keeping both the Bogolyubov coefficients α and β in the analysis. Our work also demonstrates a technique of finding out the constraint relations between the complex parameters α and β using the theoretical normalization condition and observational data of power spectrum and bispectrum from CMB.

It was quite clear right from the beginning that, because one can never determine four parameters from three pieces of information, one cannot determine the initial vacuum (i.e., the set $\{a_1, a_2, b_1, b_2\}$, each pair representing one Bogolyubov coefficient) conclusively from CMB data alone. In this article, we did not even try to do that. What we intended to do is rather to look at how far one can go in this direction using CMB. We tried to justify that one can at best give an allowed region of the parameters representing initial vacuum by utilizing the present uncertainties in the determination of f_{NL} , and that the Bunch-Davies and quite a handful of non-Bunch-Davies vacua are equally likely so far as latest CMB data is concerned. In principle, the analysis can also be subject to the next generation surveys with possibly reduced error bars for f_{NL} and a possible detection of primordial non-Gaussinities can be the most optimistic scenario.

The primary limitation of the analysis is self-explanatory. Due to less number of information than the number of unknowns, we could not determine the individual errors in the parameters a_1, a_2, b_1, b_2 . Individual errors in those parameters can be obtained only when we have a supplementary piece of information, either from CMB or from an altogether different observation. This one can do by cross-correlating CMB data with other existing or

forthcoming datasets. We are already in the process of exploring one such idea [38] and a rigorous numerical analysis in this direction is in order. We hope to address some of these issues in due course.

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7 Appendix A

Here we show the detailed calculations of three point correlation function for a generic vacuum using Green's function method. The final results have been used in Section 3 for calculation of bispectrum.

The leading order contribution in the three point correlation function from the interaction Hamiltonian is

$$\langle 0(t) | \zeta_n(t, \vec{x}) \zeta_n(t, \vec{y}) f(\zeta_n(t, \vec{z})) | 0(t) \rangle = \langle 0 | \zeta_n(t, \vec{x}) \zeta_n(t, \vec{y}) f(\zeta_n(t, \vec{z})) | 0 \rangle + \dots$$

$$|0\rangle = |0(-\infty)\rangle \Rightarrow \text{Initial state}$$

Now,

$$\begin{aligned} & \langle 0 | \zeta_n(t, \vec{x}) \zeta_n(t, \vec{y}) f(\zeta_n(t, \vec{z})) | 0 \rangle \\ &= \left[\frac{1}{2} \frac{\ddot{\phi}}{\dot{\phi}\dot{\rho}} + \frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^2} \right] \langle 0 | \zeta_n(t, \vec{x}) \zeta_n(t, \vec{y}) \zeta_n(t, \vec{z}) \zeta_n(t, \vec{z}) | 0 \rangle \\ &+ \frac{1}{\dot{\rho}} \langle 0 | \zeta_n(t, \vec{x}) \zeta_n(t, \vec{y}) \dot{\zeta}_n(t, \vec{z}) \zeta_n(t, \vec{z}) | 0 \rangle \\ &- \frac{1}{4} \frac{e^{-2\rho}}{\dot{\rho}^2} \langle 0 | \zeta_n(t, \vec{x}) \zeta_n(t, \vec{y}) \partial_k \zeta_n(t, \vec{z}) \partial^k \zeta_n(t, \vec{z}) | 0 \rangle \\ &+ \frac{1}{4} \frac{e^{-2\rho}}{\dot{\rho}^2} \langle 0 | \zeta_n(t, \vec{x}) \zeta_n(t, \vec{y}) \partial^{-2} \partial_k \partial_l (\partial^k \zeta_n(t, \vec{z}) \partial^l \zeta_n(t, \vec{z})) | 0 \rangle \\ &+ \frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^3} \langle 0 | \zeta_n(t, \vec{x}) \zeta_n(t, \vec{y}) \partial_k \zeta_n(t, \vec{z}) \partial^k \dot{\zeta}_n(t, \vec{z}) | 0 \rangle \\ &- \frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^3} \langle 0 | \zeta_n(t, \vec{x}) \zeta_n(t, \vec{y}) \partial^{-2} \partial_k \partial_l (\partial^k \zeta_n(t, \vec{z}) \partial^l \dot{\zeta}_n(t, \vec{z})) | 0 \rangle \end{aligned}$$

$$\text{And, } \overline{\zeta_n(t, \vec{x}) \zeta_n(t, \vec{z})} = G^>(t, \vec{x}; t, \vec{z}) = G^<(t, \vec{x}; t, \vec{z})$$

Therefore,

$$\begin{aligned} & \langle 0 | \zeta_n(t, \vec{x}) \zeta_n(t, \vec{y}) f(\zeta_n(t, \vec{z})) | 0 \rangle \\ &= \int \frac{d^3 \vec{k}}{(2\pi)^3} \frac{d^3 \vec{m}}{(2\pi)^3} e^{i\vec{k} \cdot (\vec{x} - \vec{z})} e^{i\vec{m} \cdot (\vec{y} - \vec{z})} \end{aligned}$$

$$\begin{aligned}
& \{ [\frac{\ddot{\phi}}{\dot{\phi}\dot{\rho}} + \frac{1}{2} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^2}] G_k^>(t, t) G_m^>(t, t) + \frac{1}{\dot{\rho}} [\dot{G}_k^>(t, t) G_m^>(t, t) + G_k^>(t, t) \dot{G}_m^>(t, t)] \\
& + \frac{1}{2} \frac{e^{-2\rho}}{\dot{\rho}^2} (\vec{k} \cdot \vec{m}) G_k^>(t, t) G_m^>(t, t) - \frac{1}{2} \frac{e^{-2\rho}}{\dot{\rho}^2} \frac{(k^2 + \vec{k} \cdot \vec{m})(m^2 + \vec{k} \cdot \vec{m})}{|\vec{k} + \vec{m}|^2} G_k^>(t, t) G_m^>(t, t) \\
& + \frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^3} [\frac{\vec{k} \cdot \vec{m}}{m^2} G_k^>(t, t) \dot{G}_m^>(t, t) + \frac{\vec{k} \cdot \vec{m}}{k^2} \dot{G}_k^>(t, t) G_m^>(t, t)] \\
& - \frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^3} \frac{(k^2 + \vec{k} \cdot \vec{m})(m^2 + \vec{k} \cdot \vec{m})}{|\vec{k} + \vec{m}|^2} [\frac{1}{m^2} G_k^>(t, t) \dot{G}_m^>(t, t) + \frac{1}{k^2} \dot{G}_k^>(t, t) G_m^>(t, t)] \}
\end{aligned}$$

Now, the three point function of the shifted field ζ_n from Schwinger-Keldysh formalism,

$$\begin{aligned}
& \langle 0(t) | \zeta_n(t, \vec{x}) \zeta_n(t, \vec{y}) \zeta_n(t, \vec{z}) | 0(t) \rangle = \langle 0 | T (\zeta_n^+(t, \vec{x}) \zeta_n^+(t, \vec{y}) \zeta_n^+(t, \vec{z}) e^{-i \int_{-\infty}^t dt' [H_I^+(t') - H_I^-(t')]}) | 0 \rangle \\
& = -i \int_{-\infty}^t dt' \langle 0 | T (\zeta_n^+(t, \vec{x}) \zeta_n^+(t, \vec{y}) \zeta_n^+(t, \vec{z}) [H_I^+(t') - H_I^-(t')]) | 0 \rangle + \dots \text{only leading order}
\end{aligned}$$

$$H_I(t) = - \int d^3 \vec{x} \{ \emptyset_1(t, \vec{x}) + \emptyset_2(t, \vec{x}) + \emptyset_3(t, \vec{x}) + \dots \}$$

Where,

$$\begin{aligned}
\emptyset_1 &= \frac{1}{4} e^{3\rho} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^4}{\dot{\rho}^4} \dot{\zeta}_n^2 \zeta_n \\
\emptyset_2 &= \frac{1}{4} e^\rho \frac{1}{M_{pl}^2} \frac{\dot{\phi}^4}{\dot{\rho}^4} \zeta_n \partial_k \zeta_n \partial^k \zeta_n \\
\emptyset_3 &= -\frac{1}{2} e^{3\rho} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^4}{\dot{\rho}^4} \dot{\zeta}_n \partial_k \zeta_n \partial^k (\partial^{-2} \dot{\zeta}_n) \\
\langle \emptyset_i(t) \rangle &= i \int_{-\infty}^t dt' \int d^3 \vec{w} \langle 0 | T (\zeta_n^+(t, \vec{x}) \zeta_n^+(t, \vec{y}) \zeta_n^+(t, \vec{z}) [\emptyset_i^+(t', \vec{w}) - \emptyset_i^-(t', \vec{w})]) | 0 \rangle \\
\langle \emptyset_1(t) \rangle &= \frac{i}{2} \frac{1}{M_{pl}^2} \int_{-\infty}^t dt' e^{3\rho(t')} \frac{\dot{\phi}^4}{\dot{\rho}^4} \int \frac{d^3 \vec{k}}{(2\pi)^3} \frac{d^3 \vec{m}}{(2\pi)^3} \frac{d^3 \vec{n}}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} e^{i\vec{m} \cdot \vec{y}} e^{i\vec{n} \cdot \vec{z}} (2\pi)^3 \delta^3(\vec{k} + \vec{m} + \vec{n}) \\
& \quad \{ \dot{G}_k^>(t, t') \dot{G}_m^>(t, t') G_n^>(t, t') - \dot{G}_k^<(t, t') \dot{G}_m^<(t, t') G_n^<(t, t') \\
& \quad + \dot{G}_k^>(t, t') G_m^>(t, t') \dot{G}_n^>(t, t') - \dot{G}_k^<(t, t') G_m^<(t, t') \dot{G}_n^<(t, t') \\
& \quad + G_k^>(t, t') \dot{G}_m^>(t, t') \dot{G}_n^>(t, t') - G_k^<(t, t') \dot{G}_m^<(t, t') \dot{G}_n^<(t, t') \} \\
\langle \emptyset_2(t) \rangle &= -\frac{i}{2} \int_{-\infty}^t dt' e^{\rho(t')} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^4}{\dot{\rho}^4} \int \frac{d^3 \vec{k}}{(2\pi)^3} \frac{d^3 \vec{m}}{(2\pi)^3} \frac{d^3 \vec{n}}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} e^{i\vec{m} \cdot \vec{y}} e^{i\vec{n} \cdot \vec{z}} (2\pi)^3 \delta^3(\vec{k} + \vec{m} + \vec{n}) \\
& \quad [\vec{k} \cdot \vec{m} + \vec{k} \cdot \vec{n} + \vec{m} \cdot \vec{n}] \{ G_k^>(t, t') G_m^>(t, t') G_n^>(t, t') - G_k^<(t, t') G_m^<(t, t') G_n^<(t, t') \} \\
\langle \emptyset_3(t) \rangle &= -\frac{i}{2} \frac{1}{M_{pl}^2} \int_{-\infty}^t dt' e^{3\rho(t')} \frac{\dot{\phi}^4}{\dot{\rho}^4} \int \frac{d^3 \vec{k}}{(2\pi)^3} \frac{d^3 \vec{m}}{(2\pi)^3} \frac{d^3 \vec{n}}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} e^{i\vec{m} \cdot \vec{y}} e^{i\vec{n} \cdot \vec{z}} (2\pi)^3 \delta^3(\vec{k} + \vec{m} + \vec{n}) \\
& \quad \left\{ \left[\frac{\vec{k} \cdot \vec{n}}{k^2} + \frac{\vec{m} \cdot \vec{n}}{m^2} \right] [\dot{G}_k^>(t, t') \dot{G}_m^>(t, t') G_n^>(t, t') - \dot{G}_k^<(t, t') \dot{G}_m^<(t, t') G_n^<(t, t')] \right\}
\end{aligned}$$

$$\begin{aligned}
& + \left[\frac{\vec{k} \cdot \vec{m}}{k^2} + \frac{\vec{n} \cdot \vec{m}}{n^2} \right] [\dot{G}_k^>(t, t') G_m^>(t, t') \dot{G}_n^>(t, t') - \dot{G}_k^<(t, t') G_m^<(t, t') \dot{G}_n^<(t, t')] \\
& + \left[\frac{\vec{m} \cdot \vec{k}}{m^2} + \frac{\vec{n} \cdot \vec{k}}{n^2} \right] [G_k^>(t, t') \dot{G}_m^>(t, t') \dot{G}_n^>(t, t') - G_k^<(t, t') \dot{G}_m^<(t, t') \dot{G}_n^<(t, t')]
\end{aligned}$$

Propagator or Greens function :-

Here we will have two propagators,

$$\begin{aligned}
G^>(t, \vec{x}; t', \vec{y}) &= \langle 0 | \zeta_n(t, \vec{x}) \zeta_n(t', \vec{y}) | 0 \rangle = \int \frac{d^3 \vec{k}}{(2\pi)^3} e^{i\vec{k} \cdot (\vec{x} - \vec{y})} G_k^>(t, t') && \text{when } t > t' \\
G^<(t, \vec{x}; t', \vec{y}) &= \langle 0 | \zeta_n(t', \vec{y}) \zeta_n(t, \vec{x}) | 0 \rangle = \int \frac{d^3 \vec{k}}{(2\pi)^3} e^{i\vec{k} \cdot (\vec{x} - \vec{y})} G_k^<(t, t') && \text{when } t < t' \\
|0\rangle &\longrightarrow \text{This vacuum is initial vacuum of free theory}
\end{aligned}$$

$$G^<(t, \vec{x}; t', \vec{y}) = G^>(t', \vec{y}; t, \vec{x})$$

$$G_k^<(t, t') = G_k^>(t', t) \quad \text{and} \quad G_k^<(t, t') = [G_k^>(t, t')]^*$$

$$\zeta_n(t, \vec{x}) = \frac{v(t, \vec{x})}{z}$$

$$G_k^>(t, t') = \zeta_n(k, t) \zeta_n^*(k, t') = \frac{v_k(t)}{z(t)} \frac{v_k^*(t')}{z(t')}$$

$$z^2 = 2e^{2\rho} \epsilon M_{pl}^2 \quad \text{or} \quad z = (\sqrt{2}\epsilon) e^{\rho} M_{pl}$$

$$G_k^>(t, t') = \frac{v_k(t) v_k^*(t')}{M_{pl}^2 (\sqrt{2}\epsilon) (\sqrt{2}\epsilon) e^{\rho(t)} e^{\rho(t')}} = \frac{v_k(t) v_k^*(t')}{2\epsilon M_{pl}^2 e^{\rho(t)} e^{\rho(t')}}$$

$$\begin{aligned}
&= \frac{(\epsilon - 1)^2}{2\epsilon M_{pl}^2} H^2 \tau \tau' [\sqrt{-\tau} \{ \alpha H_\nu^{(1)}(-k\tau) + \beta H_\nu^{(2)}(-k\tau) \}] [\sqrt{-\tau'} \{ \alpha H_\nu^{(1)}(-k\tau') + \beta H_\nu^{(2)}(-k\tau') \}]^* \\
&= \frac{(\epsilon - 1)^2}{2\epsilon M_{pl}^2} H^2 \tau \tau' [\sqrt{-\tau} \{ \alpha H_\nu^{(1)}(-k\tau) + \beta H_\nu^{(2)}(-k\tau) \}] [\sqrt{-\tau'} \{ \alpha^* H_\nu^{(2)}(-k\tau') + \beta^* H_\nu^{(1)}(-k\tau') \}] \\
&= \frac{(\epsilon - 1)^2}{2\epsilon M_{pl}^2} H^2 (\tau \tau')^{\frac{3}{2}} [|\alpha|^2 H_\nu^{(1)}(z) H_\nu^{(2)}(y) + |\beta|^2 H_\nu^{(1)}(y) H_\nu^{(2)}(z) + \alpha^* \beta H_\nu^{(2)}(z) H_\nu^{(2)}(y) + \beta^* \alpha H_\nu^{(1)}(z) H_\nu^{(1)}(y)]
\end{aligned}$$

$$\text{where,} \quad z = -k\tau \quad y = -k\tau'$$

$$H_\nu^{(1)}(z) H_\nu^{(2)}(y) = \frac{e^{iz} e^{-in\frac{\pi}{2}}}{\sqrt{\pi} \sqrt{z}} (1 - i) \left[1 + \frac{i}{8} (4\nu^2 - 1) \frac{1}{z} \right] * \frac{e^{-iy} e^{in\frac{\pi}{2}}}{\sqrt{\pi} \sqrt{y}} (1 + i) \left[1 - \frac{i}{8} (4\nu^2 - 1) \frac{1}{y} \right]$$

$$= \frac{2e^{i(z-y)}}{\pi \sqrt{zy}} \left[1 + \frac{i}{8} (4\nu^2 - 1) \left(\frac{1}{z} - \frac{1}{y} \right) + \frac{(4\nu^2 - 1)^2}{64zy} \right]$$

$$H_\nu^{(1)}(y) H_\nu^{(2)}(z) = \frac{2e^{i(y-z)}}{\pi \sqrt{zy}} \left[1 + \frac{i}{8} (4\nu^2 - 1) \left(\frac{1}{y} - \frac{1}{z} \right) + \frac{(4\nu^2 - 1)^2}{64zy} \right]$$

$$H_\nu^{(2)}(z) H_\nu^{(2)}(y) = \frac{2ie^{-i(y+z)} e^{in\pi}}{\pi \sqrt{zy}} \left[1 - \frac{i}{8} (4\nu^2 - 1) \left(\frac{1}{y} + \frac{1}{z} \right) - \frac{(4\nu^2 - 1)^2}{64zy} \right]$$

$$\begin{aligned}
H_\nu^{(1)}(z)H_\nu^{(1)}(y) &= \frac{-2ie^{i(y+z)}e^{-in\pi}}{\pi\sqrt{zy}} \left[1 + \frac{i}{8}(4\nu^2 - 1) \left(\frac{1}{y} + \frac{1}{z} \right) - \frac{(4\nu^2 - 1)^2}{64zy} \right] \\
G_k^>(t, t') &= \frac{(\epsilon - 1)^2}{2\epsilon M_{pl}^2} H^2(\tau\tau')^{\frac{3}{2}} [|\alpha|^2 H_\nu^{(1)}(z)H_\nu^{(2)}(y) + |\beta|^2 H_\nu^{(1)}(y)H_\nu^{(2)}(z) + \alpha^* \beta H_\nu^{(2)}(z)H_\nu^{(2)}(y) \\
&\quad + \beta^* \alpha H_\nu^{(1)}(z)H_\nu^{(1)}(y)] \\
&= \frac{(\epsilon - 1)^2}{k\epsilon\pi M_{pl}^2} (\tau\tau') H^2 [\{|\alpha|^2 e^{i(z-y)} + |\beta|^2 e^{i(y-z)} + ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} - ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)}\} \\
&\quad + \frac{i}{8}(4\nu^2 - 1) \frac{1}{z} \{|\alpha|^2 e^{i(z-y)} - |\beta|^2 e^{i(y-z)} - ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} - ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)}\} \\
&\quad + \frac{i}{8}(4\nu^2 - 1) \frac{1}{y} \{-|\alpha|^2 e^{i(z-y)} + |\beta|^2 e^{i(y-z)} - ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} - ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)}\} \\
&\quad + \frac{(4\nu^2 - 1)^2}{64zy} \{|\alpha|^2 e^{i(z-y)} + |\beta|^2 e^{i(y-z)} - ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} + ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)}\}] \\
G_k^>(t, t') &= \frac{(\epsilon - 1)^2}{M_{pl}^2 k\epsilon\pi} (\tau\tau') H^2 g_k^>(t, t') \\
\dot{G}_k^>(t, t') &= \frac{d}{dt'} G_k^>(t, t') = \frac{d\tau'}{dt'} \frac{d}{d\tau'} G_k^>(t, t') \\
\frac{d}{d\tau'} G_k^>(t, t') &= \frac{d}{d\tau'} \left[\frac{(\epsilon - 1)^2 (\tau\tau') H^2}{k\epsilon\pi M_{pl}^2} g_k^>(t, t') \right] = g_k^>(t, t') \frac{d}{d\tau'} \left[\frac{(\epsilon - 1)^2 \tau\tau' H^2}{k\epsilon\pi M_{pl}^2} \right] + \\
&\quad \frac{(\epsilon - 1)^2 (\tau\tau') H^2}{k\epsilon\pi M_{pl}^2} \frac{d}{d\tau'} [g_k^>(t, t')]
\end{aligned}$$

Again we have, $H = \frac{1}{(\epsilon-1)} \frac{1}{e^{\rho\tau}}$ or, $\frac{dH}{d\tau} = \frac{H\epsilon}{(1-\epsilon)\tau}$ and $\frac{d\tau}{dt} = (\epsilon - 1)H\tau$

$$\therefore \frac{d}{d\tau'} \left[\frac{(\epsilon - 1)^2 \tau\tau' H^2}{k\epsilon\pi M_{pl}^2} \right] = \frac{(1 - \epsilon^2) H^2 \tau}{k\epsilon\pi M_{pl}^2}$$

Now,

$$\begin{aligned}
\frac{d}{d\tau'} g_k^>(t, t') &= (-k) \frac{d}{dy} g_k^>(t, t') \\
&= (-k)(-i) [\{|\alpha|^2 e^{i(z-y)} - |\beta|^2 e^{i(y-z)} + ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} + ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)}\} \\
&\quad + \frac{i}{8}(4\nu^2 - 1) \frac{1}{z} \{|\alpha|^2 e^{i(z-y)} + |\beta|^2 e^{i(y-z)} - ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} + ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)}\} \\
&\quad + \frac{1}{8}(4\nu^2 - 1) \frac{1}{y^2} \{-|\alpha|^2 e^{i(z-y)} + |\beta|^2 e^{i(y-z)} - ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} - ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)}\} \\
&\quad + \frac{i}{8}(4\nu^2 - 1) \frac{1}{y} \{-|\alpha|^2 e^{i(z-y)} - |\beta|^2 e^{i(y-z)} - ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} + ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)}\} \\
&\quad + \frac{(4\nu^2 - 1)^2}{64zy} \{|\alpha|^2 e^{i(z-y)} - |\beta|^2 e^{i(y-z)} - ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} - ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)}\}
\end{aligned}$$

$$-\frac{(4\nu^2-1)^2 i}{64zy^2} \{ |\alpha|^2 e^{i(z-y)} + |\beta|^2 e^{i(y-z)} - ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} + ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)} \}$$

$$\begin{aligned} \dot{G}_k^>(t, t') &= \frac{d\tau'}{dt'} \frac{d}{d\tau'} G_k^>(t, t') \\ &= \frac{(\epsilon-1)(1-\epsilon^2)H^3\tau\tau'}{k\epsilon\pi M_{pl}^2} [\{ |\alpha|^2 e^{i(z-y)} + |\beta|^2 e^{i(y-z)} + ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} - ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)} \} \\ &\quad + \frac{i}{8}(4\nu^2-1)\frac{1}{z} \{ |\alpha|^2 e^{i(z-y)} - |\beta|^2 e^{i(y-z)} - ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} - ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)} \} \\ &\quad + \frac{i}{8}(4\nu^2-1)\frac{1}{y} \{ -|\alpha|^2 e^{i(z-y)} + |\beta|^2 e^{i(y-z)} - ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} - ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)} \} \\ &\quad + \frac{(4\nu^2-1)^2}{64zy} \{ |\alpha|^2 e^{i(z-y)} + |\beta|^2 e^{i(y-z)} - ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} + ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)} \} \\ &\quad + \frac{i(\epsilon-1)^3 H^3 \tau \tau'^2}{\epsilon \pi M_{pl}^2} [\{ |\alpha|^2 e^{i(z-y)} - |\beta|^2 e^{i(y-z)} + ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} + ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)} \} \\ &\quad + \frac{i}{8}(4\nu^2-1)\frac{1}{z} \{ |\alpha|^2 e^{i(z-y)} + |\beta|^2 e^{i(y-z)} - ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} + ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)} \} \\ &\quad + \frac{1}{8}(4\nu^2-1)\frac{1}{y^2} \{ -|\alpha|^2 e^{i(z-y)} + |\beta|^2 e^{i(y-z)} - ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} - ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)} \} \\ &\quad + \frac{i}{8}(4\nu^2-1)\frac{1}{y} \{ -|\alpha|^2 e^{i(z-y)} - |\beta|^2 e^{i(y-z)} - ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} + ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)} \} \\ &\quad + \frac{(4\nu^2-1)^2}{64zy} \{ |\alpha|^2 e^{i(z-y)} - |\beta|^2 e^{i(y-z)} - ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} - ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)} \} \\ &\quad - \frac{(4\nu^2-1)^2 i}{64zy^2} \{ |\alpha|^2 e^{i(z-y)} + |\beta|^2 e^{i(y-z)} - ie^{i\nu\pi} \alpha^* \beta e^{-i(z+y)} + ie^{-i\nu\pi} \beta^* \alpha e^{i(z+y)} \} \end{aligned}$$

$$\begin{aligned} G_k^>(t, t) &= \frac{(\epsilon-1)^2 \tau^2 H^2}{k\epsilon\pi M_{pl}^2} [\{ |\alpha|^2 + |\beta|^2 + ie^{i\nu\pi} \alpha^* \beta e^{2ik\tau} - ie^{-i\nu\pi} \beta^* \alpha e^{-2ik\tau} \} - \\ &\quad \frac{(4\nu^2-1)}{4k\tau} \{ e^{i\nu\pi} \alpha^* \beta e^{2ik\tau} + e^{-i\nu\pi} \beta^* \alpha e^{-2ik\tau} \} + \frac{(4\nu^2-1)^2}{64k^2\tau^2} \{ |\alpha|^2 + |\beta|^2 \\ &\quad - ie^{i\nu\pi} \alpha^* \beta e^{2ik\tau} + ie^{-i\nu\pi} \beta^* \alpha e^{-2ik\tau} \}] \end{aligned}$$

At $t \rightarrow \infty$

$$\begin{aligned} \lim_{t \rightarrow \infty} G_k^>(t, t) &= \lim_{\tau \rightarrow 0} \frac{(\epsilon-1)^2 H^2}{k\epsilon\pi M_{pl}^2} [\tau^2 \{ |\alpha|^2 + |\beta|^2 + ie^{i\nu\pi} \alpha^* \beta e^{2ik\tau} - ie^{-i\nu\pi} \beta^* \alpha e^{-2ik\tau} \} \\ &\quad - \frac{(4\nu^2-1)}{4k} \tau \{ e^{i\nu\pi} \alpha^* \beta e^{2ik\tau} + e^{-i\nu\pi} \beta^* \alpha e^{-2ik\tau} \} \\ &\quad + \frac{(4\nu^2-1)^2}{64k^2} \{ |\alpha|^2 + |\beta|^2 - ie^{i\nu\pi} \alpha^* \beta e^{2ik\tau} + ie^{-i\nu\pi} \beta^* \alpha e^{-2ik\tau} \}] \end{aligned}$$

$$\lim_{t \rightarrow \infty} G_k^>(t, t) = \frac{(4\nu^2 - 1)^2 (\epsilon - 1)^2 H^2}{64k^2 k\epsilon\pi M_{pl}^2} \{|\alpha|^2 + |\beta|^2 - ie^{i\nu\pi} \alpha^* \beta + ie^{-i\nu\pi} \beta^* \alpha\}$$

$$\lim_{t \rightarrow \infty} G_k^>(t, t) = \frac{1}{k^3} \frac{H^2(4\nu^2 - 1)^2 (\epsilon - 1)^2}{64\pi\epsilon M_{pl}^2} [|\alpha|^2 + |\beta|^2 - ie^{i\nu\pi} \alpha^* \beta + ie^{-i\nu\pi} \beta^* \alpha] = \frac{A}{k^3}$$

Where,

$$A = \frac{H^2(4\nu^2 - 1)^2 (\epsilon - 1)^2}{64\pi\epsilon M_{pl}^2} [|\alpha|^2 + |\beta|^2 - ie^{i\nu\pi} \alpha^* \beta + ie^{-i\nu\pi} \beta^* \alpha]$$

$$\begin{aligned} \dot{G}_k^>(t, t) &= \frac{(\epsilon - 1)(1 - \epsilon^2)H^3\tau^2}{k\epsilon\pi M_{pl}^2} [\{|\alpha|^2 + |\beta|^2 + ie^{i\nu\pi} \alpha^* \beta e^{2ik\tau} - ie^{-i\nu\pi} \beta^* \alpha e^{-2ik\tau}\} \\ &\quad - \frac{1}{4k\tau} (4\nu^2 - 1) \{e^{i\nu\pi} \alpha^* \beta e^{2ik\tau} + e^{-i\nu\pi} \beta^* \alpha e^{-2ik\tau}\} \\ &\quad + \frac{(4\nu^2 - 1)^2}{64k^2\tau^2} \{|\alpha|^2 + |\beta|^2 - ie^{i\nu\pi} \alpha^* \beta e^{2ik\tau} + ie^{-i\nu\pi} \beta^* \alpha e^{-2ik\tau}\}] \\ &\quad + \frac{i(\epsilon - 1)^3 H^3\tau^3}{\epsilon\pi M_{pl}^2} [\{|\alpha|^2 - |\beta|^2 + ie^{i\nu\pi} \alpha^* \beta e^{2ik\tau} + ie^{-i\nu\pi} \beta^* \alpha e^{-2ik\tau}\} \\ &\quad - \frac{i(4\nu^2 - 1)}{8k\tau} \{|\alpha|^2 + |\beta|^2 - ie^{i\nu\pi} \alpha^* \beta e^{2ik\tau} + ie^{-i\nu\pi} \beta^* \alpha e^{-2ik\tau}\} \\ &\quad + \frac{1}{8k^2\tau^2} (4\nu^2 - 1) \{-|\alpha|^2 + |\beta|^2 - ie^{i\nu\pi} \alpha^* \beta e^{2ik\tau} - ie^{-i\nu\pi} \beta^* \alpha e^{-2ik\tau}\} \\ &\quad - \frac{1}{8k\tau} (4\nu^2 - 1) \{-|\alpha|^2 - |\beta|^2 - ie^{i\nu\pi} \alpha^* \beta e^{2ik\tau} + ie^{-i\nu\pi} \beta^* \alpha e^{-2ik\tau}\} \\ &\quad + \frac{(4\nu^2 - 1)^2}{64k^2\tau^2} \{|\alpha|^2 - |\beta|^2 - ie^{i\nu\pi} \alpha^* \beta e^{2ik\tau} - ie^{-i\nu\pi} \beta^* \alpha e^{-2ik\tau}\} \\ &\quad + i \frac{(4\nu^2 - 1)^2}{64k^3\tau^3} \{|\alpha|^2 + |\beta|^2 - ie^{i\nu\pi} \alpha^* \beta e^{2ik\tau} + ie^{-i\nu\pi} \beta^* \alpha e^{-2ik\tau}\}] \end{aligned}$$

$$\lim_{t \rightarrow \infty} \dot{G}_k^>(t, t) = -(\epsilon - 1)^2 \frac{H^3(4\nu^2 - 1)^2}{32\pi k^3 M_{pl}^2} [|\alpha|^2 + |\beta|^2 - i\alpha^* \beta e^{i\nu\pi} + i\beta^* \alpha e^{-i\nu\pi}] = \frac{B}{k^3}$$

Where,

$$B = -(\epsilon - 1)^2 \frac{H^3(4\nu^2 - 1)^2}{32\pi M_{pl}^2} [|\alpha|^2 + |\beta|^2 - i\alpha^* \beta e^{i\nu\pi} + i\beta^* \alpha e^{-i\nu\pi}]$$

$$\lim_{t \rightarrow \infty} G_k^>(t, t') = \frac{1}{k^3} C_1 e^{ik\tau'} + \frac{1}{k^3} C_2 e^{-ik\tau'} + \frac{1}{k^2} C_3 \tau' e^{ik\tau'} + \frac{1}{k^2} C_4 \tau' e^{-ik\tau'}$$

Where,

$$C_1 = \frac{H^2(4\nu^2 - 1)^2 (\epsilon - 1)^2}{64\pi\epsilon M_{pl}^2} [|\alpha|^2 - i\alpha^* \beta e^{i\nu\pi}]$$

$$C_2 = \frac{H^2(4\nu^2 - 1)^2 (\epsilon - 1)^2}{64\pi\epsilon M_{pl}^2} [|\beta|^2 + i\alpha \beta^* e^{-i\nu\pi}]$$

$$C_3 = \frac{-iH^2(4\nu^2 - 1)(\epsilon - 1)^2}{8\pi\epsilon M_{pl}^2} [|\alpha|^2 - i\alpha^*\beta e^{i\nu\pi}]$$

$$C_4 = \frac{-iH^2(4\nu^2 - 1)(\epsilon - 1)^2}{8\pi\epsilon M_{pl}^2} [-|\beta|^2 - i\alpha\beta^* e^{-i\nu\pi}]$$

$$\lim_{t \rightarrow \infty} \dot{G}_k^>(t, t') = \frac{1}{k^2} C_5 \tau' e^{ik\tau'} + \frac{1}{k^2} C_6 \tau' e^{-ik\tau'} + \frac{1}{k} C_7 \tau'^2 e^{ik\tau'} + \frac{1}{k} C_8 \tau'^2 e^{-ik\tau'} + \frac{1}{k^3} C_9 e^{ik\tau'} + \frac{1}{k^3} C_{10} e^{-ik\tau'}$$

$$C_5 = \{(\epsilon - 1)(4\nu^2 - 1) + 8(\epsilon + 1)\} \left\{ \frac{iH^3(4\nu^2 - 1)(\epsilon - 1)^2}{64\pi\epsilon M_{pl}^2} \right\} [|\alpha|^2 - i\alpha^*\beta e^{i\nu\pi}]$$

$$C_6 = \{(\epsilon - 1)(4\nu^2 - 1) + 8(\epsilon + 1)\} \left\{ \frac{iH^3(4\nu^2 - 1)(\epsilon - 1)^2}{64\pi\epsilon M_{pl}^2} \right\} [-|\beta|^2 - i\alpha\beta^* e^{-i\nu\pi}]$$

$$C_7 = \frac{H^3(4\nu^2 - 1)(\epsilon - 1)^3}{8\pi\epsilon M_{pl}^2} [|\alpha|^2 - i\alpha^*\beta e^{i\nu\pi}]$$

$$C_8 = \frac{H^3(4\nu^2 - 1)(\epsilon - 1)^3}{8\pi\epsilon M_{pl}^2} [|\beta|^2 + i\alpha\beta^* e^{-i\nu\pi}]$$

$$C_9 = \frac{-H^3(4\nu^2 - 1)^2(\epsilon - 1)^2}{32\pi M_{pl}^2} [|\alpha|^2 - i\alpha^*\beta e^{i\nu\pi}]$$

$$C_{10} = \frac{-H^3(4\nu^2 - 1)^2(\epsilon - 1)^2}{32\pi M_{pl}^2} [|\beta|^2 + i\alpha\beta^* e^{-i\nu\pi}]$$

$$\lim_{k\tau \rightarrow 0} G_k^>(t, t') = \frac{1}{k^3} \{C_1 e^{ik\tau'} + C_2 e^{-ik\tau'} + (k\tau') C_3 e^{ik\tau'} + (k\tau') C_4 e^{-ik\tau'}\}$$

Now for the condition $k\tau' \rightarrow \infty$,

$$\lim_{k\tau \rightarrow 0} G_k^>(t, t') = \frac{1}{k^3} (k\tau') \{C_3 e^{ik\tau'} + C_4 e^{-ik\tau'}\}$$

$$\lim_{t \rightarrow \infty} \dot{G}_k^>(t, t') = \frac{1}{k^2} C_5 \tau' e^{ik\tau'} + \frac{1}{k^2} C_6 \tau' e^{-ik\tau'} + \frac{1}{k} C_7 \tau'^2 e^{ik\tau'} + \frac{1}{k} C_8 \tau'^2 e^{-ik\tau'} + \frac{1}{k^3} C_9 e^{ik\tau'} + \frac{1}{k^3} C_{10} e^{-ik\tau'}$$

$$\lim_{t \rightarrow \infty} \dot{G}_k^>(t, t') = \frac{1}{k^3} \{k C_5 \tau' e^{ik\tau'} + k C_6 \tau' e^{-ik\tau'} + k^2 C_7 \tau'^2 e^{ik\tau'} + k^2 C_8 \tau'^2 e^{-ik\tau'} + C_9 e^{ik\tau'} + C_{10} e^{-ik\tau'}\}$$

$$\lim_{k\tau \rightarrow 0} \dot{G}_k^>(t, t') = \frac{1}{k^3} \{(k\tau') C_5 e^{ik\tau'} + (k\tau') C_6 e^{-ik\tau'} + (k\tau')^2 C_7 e^{ik\tau'} + (k\tau')^2 C_8 e^{-ik\tau'} + C_9 e^{ik\tau'} + C_{10} e^{-ik\tau'}\}$$

Now for the condition $k\tau' \rightarrow \infty$,

$$\lim_{k\tau \rightarrow 0} \dot{G}_k^>(t, t') = \frac{1}{k^3} \{(k\tau')^2 C_7 e^{ik\tau'} + (k\tau')^2 C_8 e^{-ik\tau'}\}$$

We have used these results in Section 3 and subsequent sections.

8 Appendix B

This section is dedicated to a detailed derivation of Bispectrum for a generic vacuum. The results have been used in Section 3.

The contributions from $f(\zeta_n)$ terms,

$$\begin{aligned}
& \langle 0 | \zeta_n(t, \vec{x}) \zeta_n(t, \vec{y}) f(\zeta_n(t, \vec{z})) | 0 \rangle \\
&= \int \frac{d^3 \vec{k}}{(2\pi)^3} \frac{d^3 \vec{m}}{(2\pi)^3} e^{i\vec{k} \cdot (\vec{x} - \vec{z})} e^{i\vec{m} \cdot (\vec{y} - \vec{z})} \left\{ \left[\frac{\ddot{\phi}}{\dot{\phi}\dot{\rho}} + \frac{1}{2} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^2} \right] G_k^>(t, t) G_m^>(t, t) + \frac{1}{\dot{\rho}} [\dot{G}_k^>(t, t) G_m^>(t, t) \right. \right. \\
&+ G_k^>(t, t) \dot{G}_m^>(t, t)] + \frac{1}{2} \frac{e^{-2\rho}}{\dot{\rho}^2} (\vec{k} \cdot \vec{m}) G_k^>(t, t) G_m^>(t, t) - \frac{1}{2} \frac{e^{-2\rho}}{\dot{\rho}^2} \frac{(k^2 + \vec{k} \cdot \vec{m})(m^2 + \vec{k} \cdot \vec{m})}{|\vec{k} + \vec{m}|^2} G_k^>(t, t) G_m^>(t, t) \\
&+ \frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^3} \left[\frac{\vec{k} \cdot \vec{m}}{m^2} G_k^>(t, t) \dot{G}_m^>(t, t) + \frac{\vec{k} \cdot \vec{m}}{k^2} \dot{G}_k^>(t, t) G_m^>(t, t) \right] \\
&\left. - \frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^3} \frac{(k^2 + \vec{k} \cdot \vec{m})(m^2 + \vec{k} \cdot \vec{m})}{|\vec{k} + \vec{m}|^2} \left[\frac{1}{m^2} G_k^>(t, t) \dot{G}_m^>(t, t) + \frac{1}{k^2} \dot{G}_k^>(t, t) G_m^>(t, t) \right] \right\}
\end{aligned}$$

Now,

$$e^{-2\rho(t)} = H^2(\epsilon - 1)^2 \tau^2$$

Now at $\tau \longrightarrow 0$ or, $t \longrightarrow \infty$ $e^{-2\rho(t)} \longrightarrow 0$

Therefore, 3rd and 4th term will not contribute,

1st term,

$$\lim_{t \rightarrow \infty} G_k^>(t, t) G_m^>(t, t) = \frac{A}{k^3} \frac{A}{m^3} = \frac{A^2}{k^3 m^3}$$

2nd term,

$$\dot{G}_k^>(t, t) G_m^>(t, t) + G_k^>(t, t) \dot{G}_m^>(t, t) = \frac{B}{k^3} \frac{A}{m^3} + \frac{A}{k^3} \frac{B}{m^3} = \frac{2AB}{k^3 m^3}$$

5th term,

$$\frac{\vec{k} \cdot \vec{m}}{m^2} G_k^>(t, t) \dot{G}_m^>(t, t) + \frac{\vec{k} \cdot \vec{m}}{k^2} \dot{G}_k^>(t, t) G_m^>(t, t) = \frac{\vec{k} \cdot \vec{m}}{m^2} \frac{A}{k^3} \frac{B}{m^3} + \frac{\vec{k} \cdot \vec{m}}{k^2} \frac{B}{k^3} \frac{A}{m^3} = \frac{AB}{k^3 m^3} \left(\frac{1}{k^2} + \frac{1}{m^2} \right) \vec{k} \cdot \vec{m}$$

6th term,

$$\begin{aligned}
& \frac{1}{m^2} G_k^>(t, t) \dot{G}_m^>(t, t) + \frac{1}{k^2} \dot{G}_k^>(t, t) G_m^>(t, t) = \frac{1}{m^2} \frac{A}{k^3} \frac{B}{m^3} + \frac{1}{k^2} \frac{B}{k^3} \frac{A}{m^3} = \frac{AB}{k^3 m^3} \left(\frac{1}{k^2} + \frac{1}{m^2} \right) \\
&= \int \frac{d^3 \vec{k}}{(2\pi)^3} \frac{d^3 \vec{m}}{(2\pi)^3} e^{(i\vec{k} \cdot \vec{x})} e^{(i\vec{m} \cdot \vec{y})} e^{-i(\vec{k} + \vec{m}) \cdot \vec{z}} \left\{ \left[\frac{\ddot{\phi}}{\dot{\phi}\dot{\rho}} + \frac{1}{2} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^2} \right] \frac{A^2}{k^3 m^3} + \frac{1}{\dot{\rho}} \left[\frac{2AB}{k^3 m^3} \right] \right. \\
&+ \frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^3} \left[\frac{\vec{k} \cdot \vec{m}}{k^3 m^3} AB \right] \left[\frac{1}{k^2} + \frac{1}{m^2} \right] - \frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^3} \frac{(k^2 + \vec{k} \cdot \vec{m})(m^2 + \vec{k} \cdot \vec{m})}{|\vec{k} + \vec{m}|^2} \left[\frac{1}{k^2} + \frac{1}{m^2} \right] \frac{AB}{k^3 m^3} \Big\} \\
&+ \text{two more} \\
&= \int \frac{d^3 \vec{k}}{(2\pi)^3} \frac{d^3 \vec{m}}{(2\pi)^3} \frac{d^3 \vec{n}}{(2\pi)^3} e^{(i\vec{k} \cdot \vec{x})} e^{(i\vec{m} \cdot \vec{y})} e^{(i\vec{n} \cdot \vec{z})} (2\pi)^3 \delta^3(\vec{k} + \vec{m} + \vec{n})
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{k^3 m^3 n^3} \left[\left(\frac{\ddot{\phi}}{\dot{\phi}\dot{\rho}} + \frac{1}{2} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^2} \right) (k^3 + m^3 + n^3) \right] A^2 + \left[\frac{2}{\dot{\rho}} (k^3 + m^3 + n^3) \right. \\
& + \frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^3} \frac{1}{k^2 m^2 n^2} \{ (k^2 + m^2) (\vec{k} \cdot \vec{m}) n^5 + (k^2 + n^2) (\vec{k} \cdot \vec{n}) m^5 + (m^2 + n^2) (\vec{m} \cdot \vec{n}) k^5 \} \\
& - \frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^3} \frac{1}{k^2 m^2 n^2} \left\{ \frac{(k^2 + \vec{k} \cdot \vec{m})(m^2 + \vec{k} \cdot \vec{m})}{|\vec{k} + \vec{m}|^2} (k^2 + m^2) n^5 \right. \\
& \left. + \frac{(k^2 + \vec{k} \cdot \vec{n})(n^2 + \vec{k} \cdot \vec{n})}{|\vec{k} + \vec{n}|^2} (k^2 + n^2) m^5 + \frac{(m^2 + \vec{m} \cdot \vec{n})(n^2 + \vec{m} \cdot \vec{n})}{|\vec{m} + \vec{n}|^2} (m^2 + n^2) k^5 \right\} \left. \right] AB
\end{aligned}$$

$$\langle \emptyset_1(t) \rangle = \int \left\{ \frac{i}{M_{pl}^2} \frac{\dot{\phi}^4}{\dot{\rho}^4} \frac{1}{H^4(\epsilon - 1)^4} \right\} \frac{d^3 \vec{k}}{(2\pi)^3} \frac{d^3 \vec{m}}{(2\pi)^3} \frac{d^3 \vec{n}}{(2\pi)^3} e^{(i\vec{k} \cdot \vec{x})} e^{(i\vec{m} \cdot \vec{y})} e^{(i\vec{n} \cdot \vec{z})} (2\pi)^3 \delta^3(\vec{k} + \vec{m} + \vec{n})$$

$$\begin{aligned}
& \left[\frac{1}{k^2 m^2 n^2} \left\{ D_1 \frac{km + kn + mn}{(k + m + n)^2} + D_2 \left\{ \frac{km}{(k + m - n)^2} + \frac{kn}{(k + n - m)^2} + \frac{mn}{(m + n - k)^2} \right\} \right. \right. \\
& + (D_3 + D_4) \left\{ \frac{km}{(m + n - k)^2} + \frac{km}{(k + n - m)^2} + \frac{kn}{(m + n - k)^2} + \frac{kn}{(k + m - n)^2} + \frac{mn}{(k + m - n)^2} \right. \\
& \left. \left. + \frac{mn}{(k + n - m)^2} \right\} \right\} \left. \right]
\end{aligned}$$

$$\begin{aligned}
\langle \emptyset_2(t) \rangle &= \int \left\{ -\frac{i}{M_{pl}^2} \frac{\dot{\phi}^4}{\dot{\rho}^4} \frac{1}{H^2(\epsilon - 1)^2} \right\} \left(\frac{\vec{k} \cdot \vec{m} + \vec{k} \cdot \vec{n} + \vec{m} \cdot \vec{n}}{k^2 m^2 n^2} \right) \frac{d^3 \vec{k}}{(2\pi)^3} \frac{d^3 \vec{m}}{(2\pi)^3} \frac{d^3 \vec{n}}{(2\pi)^3} e^{(i\vec{k} \cdot \vec{x})} e^{(i\vec{m} \cdot \vec{y})} e^{(i\vec{n} \cdot \vec{z})} (2\pi)^3 \\
& \delta^3(\vec{k} + \vec{m} + \vec{n}) \left[\left\{ (C_3^3 + C_4^3) \frac{1}{(k + m + n)^2} \right\} + (C_3^2 C_4 + C_4^2 C_3) \left\{ \frac{1}{(-k + m + n)^2} + \frac{1}{(k - m + n)^2} + \frac{1}{(k + m - n)^2} \right\} \right]
\end{aligned}$$

$$\langle \emptyset_3(t) \rangle = \int \left\{ -\frac{i}{M_{pl}^2} \frac{\dot{\phi}^4}{\dot{\rho}^4} \frac{1}{H^4(\epsilon - 1)^4} \frac{1}{k^2 m^2 n^2} \right\} \frac{d^3 \vec{k}}{(2\pi)^3} \frac{d^3 \vec{m}}{(2\pi)^3} \frac{d^3 \vec{n}}{(2\pi)^3} e^{(i\vec{k} \cdot \vec{x})} e^{(i\vec{m} \cdot \vec{y})} e^{(i\vec{n} \cdot \vec{z})} (2\pi)^3 \delta^3(\vec{k} + \vec{m} + \vec{n})$$

$$\begin{aligned}
& [D_1 \left\{ \frac{m^2(\vec{k} \cdot \vec{n}) + k^2(\vec{m} \cdot \vec{n})}{km} \frac{1}{(k + m + n)^2} + \frac{n^2(\vec{k} \cdot \vec{m}) + k^2(\vec{n} \cdot \vec{m})}{kn} \frac{1}{(k + m + n)^2} \right. \\
& \left. + \frac{n^2(\vec{m} \cdot \vec{k}) + m^2(\vec{n} \cdot \vec{k})}{mn} \frac{1}{(k + m + n)^2} \right\} \\
& + D_2 \left\{ \frac{m^2(\vec{k} \cdot \vec{n}) + k^2(\vec{m} \cdot \vec{n})}{km} \frac{1}{(k + m - n)^2} + \frac{n^2(\vec{k} \cdot \vec{m}) + k^2(\vec{n} \cdot \vec{m})}{kn} \frac{1}{(k + n - m)^2} \right. \\
& \left. + \frac{n^2(\vec{m} \cdot \vec{k}) + m^2(\vec{n} \cdot \vec{k})}{mn} \frac{1}{(m + n - k)^2} \right\} \\
& + (D_3 + D_4) \left\{ \frac{m^2(\vec{k} \cdot \vec{n}) + k^2(\vec{m} \cdot \vec{n})}{km} \left(\frac{1}{(m + n - k)^2} + \frac{1}{(k + n - m)^2} \right) \right.
\end{aligned}$$

$$+ \frac{n^2(\vec{k} \cdot \vec{m}) + k^2(\vec{n} \cdot \vec{m})}{kn} \left(\frac{1}{(m+n-k)^2} + \frac{1}{(k+m-n)^2} \right) \\ + \frac{n^2(\vec{m} \cdot \vec{k}) + m^2(\vec{n} \cdot \vec{k})}{mn} \left(\frac{1}{(k+m-n)^2} + \frac{1}{(k+n-m)^2} \right) \}]]$$

$$D_1 = C_3 C_7^2 + C_4 C_8^2, \quad D_2 = C_3 C_8^2 + C_4 C_7^2, \quad D_3 = C_3 C_7 C_8, \quad D_4 = C_4 C_7 C_8$$

Final form of the Bispectrum

$$\int \frac{d^3 \vec{k}}{(2\pi)^3} \frac{d^3 \vec{m}}{(2\pi)^3} \frac{d^3 \vec{n}}{(2\pi)^3} e^{i(\vec{k} \cdot \vec{x} + \vec{m} \cdot \vec{y} + \vec{n} \cdot \vec{z})} (2\pi)^3 \delta^3(\vec{k} + \vec{m} + \vec{n}) \frac{1}{k^3 m^3 n^3} \frac{H^4}{32\epsilon^2 M_{pl}^4} \\ [\{ (\frac{\ddot{\phi}}{\dot{\phi}\dot{\rho}} + \frac{1}{2} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^2}) A^2 + \frac{2}{\dot{\rho}} AB - \frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^3} AB + (-i \frac{2\epsilon^2 M_{pl}^2}{H^4(1-\epsilon)^4}) D_1 \} \frac{32\epsilon^2 M_{pl}^4}{H^4}] (k^3 + m^3 + n^3) \\ + [\frac{1}{4} \frac{1}{M_{pl}^2} \frac{\dot{\phi}^2}{\dot{\rho}^3} \frac{32\epsilon^2 M_{pl}^4}{H^4} AB \{ \frac{(k^3 - m^3)(k^2 - m^2)}{2n^2} + \frac{(k^3 - n^3)(k^2 - n^2)}{2m^2} + \frac{(m^3 - n^3)(m^2 - n^2)}{2k^2} \\ - \frac{k}{4} (k^2 + m^2 - n^2)(k^2 - m^2 + n^2) (\frac{1}{m^2} + \frac{1}{n^2}) - \frac{m}{4} (k^2 + m^2 - n^2)(m^2 - k^2 + n^2) (\frac{1}{k^2} + \frac{1}{n^2}) \\ - \frac{n}{4} (k^2 + n^2 - m^2)(m^2 - k^2 + n^2) (\frac{1}{k^2} + \frac{1}{m^2}) \} + [\frac{i128\epsilon^4 M_{pl}^6}{(1-\epsilon)^4 H^8}] [\frac{kmn(km + kn + mn)}{(k+m+n)^2} D_1 \\ + \{ \frac{km}{(k+m-n)^2} + \frac{kn}{(k+n-m)^2} + \frac{mn}{(m+n-k)^2} \} kmn D_2 + \{ \frac{km}{(m+n-k)^2} + \frac{km}{(k+n-m)^2} \\ + \frac{kn}{(m+n-k)^2} + \frac{kn}{(k+m-n)^2} + \frac{mn}{(k+m-n)^2} + \frac{mn}{(k+n-m)^2} \} kmn (D_3 + D_4)] \\ + [-i \frac{128\epsilon^4 M_{pl}^6}{(1-\epsilon)^2 H^6}] [\{ -\frac{1}{2} \frac{(k^2 + m^2 + n^2)}{(k+m+n)} \} kmn (C_3^3 + C_4^3) - \frac{1}{2} \{ \frac{(k^2 + m^2 + n^2)}{(k+m-n)^2} + \frac{(k^2 + m^2 + n^2)}{(k+n-m)^2} \\ + \frac{(k^2 + m^2 + n^2)}{(m+n-k)^2} \} C_3 C_4 (C_3 + C_4) kmn] + [-\frac{i128\epsilon^4 M_{pl}^6}{(1-\epsilon)^4 H^8}] [-\{ \frac{1}{2} \frac{(k^4 + m^4 + n^4)}{(k+m+n)} \\ + \frac{k^2 m^2 + k^2 n^2 + m^2 n^2}{(k+m+n)} + kmn \frac{(k^2 + m^2 + n^2)}{(k+m+n)} \} D_1 + \{ \frac{n}{2} \frac{((k^2 - m^2)^2 - n^2(k^2 + m^2))}{(k+m-n)^2}$$

$$\begin{aligned}
& + \frac{m}{2} \frac{((k^2 - n^2)^2 - m^2(k^2 + n^2))}{(k + n - m)^2} + \frac{k}{2} \frac{((m^2 - n^2)^2 - k^2(m^2 + n^2))}{(-k + m + n)^2} \} D_2 + \{ \frac{n}{2} ((k^2 - m^2)^2 - n^2(k^2 + m^2)) \\
& (\frac{1}{(m + n - k)^2} + \frac{1}{(k + n - m)^2}) + \frac{m}{2} ((k^2 - n^2)^2 - m^2(k^2 + n^2)) (\frac{1}{(m + n - k)^2} + \frac{1}{(k + m - n)^2}) \\
& + \frac{k}{2} ((m^2 - n^2)^2 - k^2(m^2 + n^2)) (\frac{1}{(k + m - n)^2} + \frac{1}{(k + n - m)^2}) \} (D_3 + D_4)] \\
& = \int \frac{d^3 \vec{k}}{(2\pi)^3} \frac{d^3 \vec{m}}{(2\pi)^3} \frac{d^3 \vec{n}}{(2\pi)^3} e^{i(\vec{k} \cdot \vec{x} + \vec{m} \cdot \vec{y} + \vec{n} \cdot \vec{z})} (2\pi)^3 \delta^3(\vec{k} + \vec{m} + \vec{n}) \frac{1}{k^3 m^3 n^3} \frac{H^4}{32\epsilon^2 M_{pl}^4} \mathcal{A}
\end{aligned}$$

where, \mathcal{A} is defined in Section 3.

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